

SELECTED PROBLEMS OF BARYONS SPECTROSCOPY: CHIRAL SOLITON VERSUS QUARK MODELS*

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Abstract

Inconsistency between rigid rotator and bound state models at arbitrary number of colors, rigid rotator — soft rotator dilemma and some other problems of baryon spectroscopy are discussed in the framework of the chiral soliton approach (CSA). Consequences of the comparison of CSA results with simple quark models are considered and the $1/N_c$ expansion for the effective strange antiquark mass is presented, as it follows from the CSA. Strong dependence of the effective strange antiquark mass on the SU(3) multiplet is required to fit the CSA predictions. The difference of ‘good’ and ‘bad’ diquark masses, which is about 100 Mev, is in reasonable agreement with other estimates. Multibaryons (hypernuclei) with strangeness are described and some states of interest are predicted within CSA as well.

1 Introduction

In spite of (or due to?) recent dramatic events with (non)observation of narrow pentaquark states, the studies of baryons spectrum — nonstrange, strange, and with heavy flavors — remain to be very actual for accelerator physics. Discovery of baryon states besides well established (e.g., octet, decuplet and certain resonances) could help to the progress in understanding hadrons structure.

In the absence of the complete theory of strong interactions there are different approaches and models of hadron structure; each has some advantages and certain drawbacks. Interpretation of hadrons spectra in terms of quark models (QM) is widely accepted, QM are “most successful tool for the classification and interpretation” (R.Jaffe) of hadrons spectrum. These models are so widely accepted because (probably) they correspond to our intuitive ideas how the bigger object — baryon, for example — can be made of smaller ones — quarks. However, our intuition, based on macroscopic experience, may be totally misleading in the world of elementary particles.

QM are to large extent phenomenological since there are no regular methods of solving relativistic many-body problem. The number of constituents (e.g. additional $q\bar{q}$ -pairs) and their weight should not be fixed as starting condition, in a true relativistic theory, but should be obtained by means of solving adequate relativistic equations (and the quark confinement should be obtained in this way, as well!).

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In view of this global unresolved problem alternative approaches are of interest. In particular, the chiral soliton approach (CSA) based on few principles represented by the model lagrangian, has certain advantages. Baryons and baryonic systems are considered on equal footing (the look ‘from outside’). CSA has many features of a true theory, but still it is a model: some elements of phenomenology are present necessarily in CSA as well. Results obtained within CSA mimic some features of baryons spectrum within quark models due to Gell-Mann — Okubo relations for the masses of baryons within definite $SU(3)$ multiplet.

2 Features of the chiral soliton approach

The CSA is based on fundamental principles and ingredients incorporated in the *truncated effective chiral lagrangian*:

$$L^{eff} = -\frac{F_\pi^2}{16}Tr l_\mu l_\mu + \frac{1}{32e^2}Tr[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8}Tr(U + U^\dagger - 2) + \dots, \quad (1)$$

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $\in SU(3)$ - unitary matrix depending on chiral fields, m_π is pion mass, F_π -pion decay constant taken from experiment, e - the only parameter of the model which defines the weight of the antisymmetric 4-th order in chiral derivatives term in the lagrangian (Skyrme term)¹. The effective lagrangian (1) can be deduced from underlying lagrangian of Quantum Chromodynamics [1], and in this way the infinite number of terms appear. The higher order in l_μ terms are not shown in (1). 6-th order term is taken into account in a number of calculations, and it does not change the properties of multiskyrmions considerably. The mass term $\sim F_\pi^2 m_\pi^2$, changes asymptotics of the profile f and the structure of multiskyrmions at large B . For the $SU(2)$ case

$$U = \cos f + i(\vec{n}\vec{\tau})\sin f, \quad (2)$$

unit vector \vec{n} depends on 2 functions, α , β . Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parametrize the unit vector on the 3-sphere S^3 .

The soliton is configuration of chiral fields, possessing topological charge identified with the baryon number B (Skyrme, 1961):

$$B = \frac{-1}{2\pi^2} \int s_f^2 s_\alpha I[(f, \alpha, \beta)/(x, y, z)] d^3r \quad (3)$$

where I is the Jacobian of the coordinates transformation, $s_f = \sin f$, $s_\alpha = \sin \alpha$, etc. So, the quantity B shows how many times S^3 is covered when integration over R^3 is made. Recall that surface of the unit sphere S^3 equals

$$\int s_f^2 s_\alpha df d\alpha d\beta = 2\pi^2. \quad (4)$$

Minimization of the classical mass functional M_{cl} for each value of baryon number provides 3 profiles f, α, β , the mass of static configuration and allows to calculate binding energies of classical configurations, moments of inertia Θ_I , Θ_J , Θ_K and some other characteristics of chiral solitons wch contain implicitly information about interaction between baryons and are necessary to perform the quantization procedure — to get the spectrum of baryon states with definite quantum numbers.

¹In some papers the constant F_π and even the mass m_π are considered as parameters, although they are fixed by existing data. Such approach is useful, however, for investigations of some global properties of chiral soliton models and multiskyrmions.

3 Skyrmions quantization and spectrum of baryons

The observed spectrum of states is obtained by means of quantization procedure and depends on quantum numbers of baryons and above mentioned properties of classical configurations, moments of inertia, Σ -term (Γ), etc. In $SU(2)$ case the rigid rotator model (RRM)[2] is most effective and successful in describing the properties of nucleons, Δ -isobar, some properties light nuclei (N.Manton, S.Krusch and S.Wood, [3]) and also the so called ‘‘symmetry energy’’ of nuclei with $A \lesssim 20$ [4].

In the $SU(3)$ case different quantization models have been developed. Probably, mostly accepted way to get the spectrum of baryons is to place the established $SU(2)$ classical configuration (e.g. the so called ‘hedgehog’ for the $B = 1$ skyrmion) in the upper left corner of the $SU(3)$ matrix of chiral fields and to quantize the $SU(3)$ zero modes corresponding to rotations in the $SU(3)$ configuration space [5]. The following mass formula takes place, corresponding to this rigid rotator model:

$$M(p, q, Y, I, J) = M_{cl} + \frac{K(p, q, I_R)}{2\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \delta M_{(p,q)}(Y, I), \quad (5)$$

$$\sim N_c \quad \sim 1 \quad \sim N_c^{-1} \quad \sim 1,$$

which is in fact expansion in powers of $1/N_c$.

$$K(p, q, I_R) = C_2(SU3) - I_R(I_R + 1) - N_c^2 B^2/12, \quad C_2(SU3) = (p^2 + q^2 + pq)/3 + p + q,$$

p, q are the numbers of upper and lower indices in the spinor describing the $SU(3)$ multiplet, I_R is the so called ‘right’ isospin, $I_R = J$ - the value of spin of the $B = 1$ state. Some paradox is in the fact that total splitting of the whole multiplet is $\sim N_c$.

Mass splittings δM are due to the term in the lagrangian

$$\mathcal{L}_M \simeq -\tilde{m}_K^2 \Gamma \frac{s_\nu^2}{2}, \quad (6)$$

ν is the angle of rotation into ‘strange’ direction, $\tilde{m}_K^2 = F_K^2 m_K^2 / F_\pi^2 - m_\pi^2$ includes $SU(3)$ -symmetry violation in flavor decay constants. For accepted values of the model parameters numerical values of some important characteristics of the $B = 1$ skyrmion are: $\Gamma \sim 5 \text{ Gev}^{-1} \sim \Sigma$, moments of inertia $\Theta_\pi \sim (5 - 6) \text{ Gev}^{-1}$, $\Theta_K \sim (2 - 3) \text{ Gev}^{-1}$. All inertia are proportional to the number of colors, $\Theta \sim N_c$.

The multiplets of exotic baryons are shown in Fig. 1. Recall that for the ‘octet’ $[p, q] = [1, (N_c - 1)/2]$, for ‘decuplet’ $[p, q] = [3, (N_c - 3)/2]$, and $(p + 2q) = N_c$. For exotic multiplets shown in Fig.1 $p + 2q = N_c + 3$ ². The lower index in notations of states indicates the isospin of the state, e.g.

$$\Phi/\Xi_{3/2} = |\overline{10}, S = -2, I = 3/2 \rangle, \quad \Sigma_2 = |27, S = -1, I = 2 \rangle, \quad \Omega_1 = |27, S = -3, I = 1 \rangle.$$

‘‘Strangeness contents’’

$$C_S = \langle s_\nu^2/2 \rangle_B \quad (7)$$

can be calculated exactly with the help of wave functions in $SU(3)$ configuration space, for arbitrary number of colors N_c [6, 7].

Some examples of values of C_S at arbitrary number of colors N_c are presented in Tables 1 and 2 taken from [7]³.

²This particular choice of $[p, q]$ values is really a result of convention for large N_c generalization of the model. For this choice the upper states within each $SU(3)$ multiplet at arbitrary N_c coincide with those at $N_c = 3$.

³For the case of ‘nucleon’ strangeness content at arbitrary N_c has been presented first in [8].

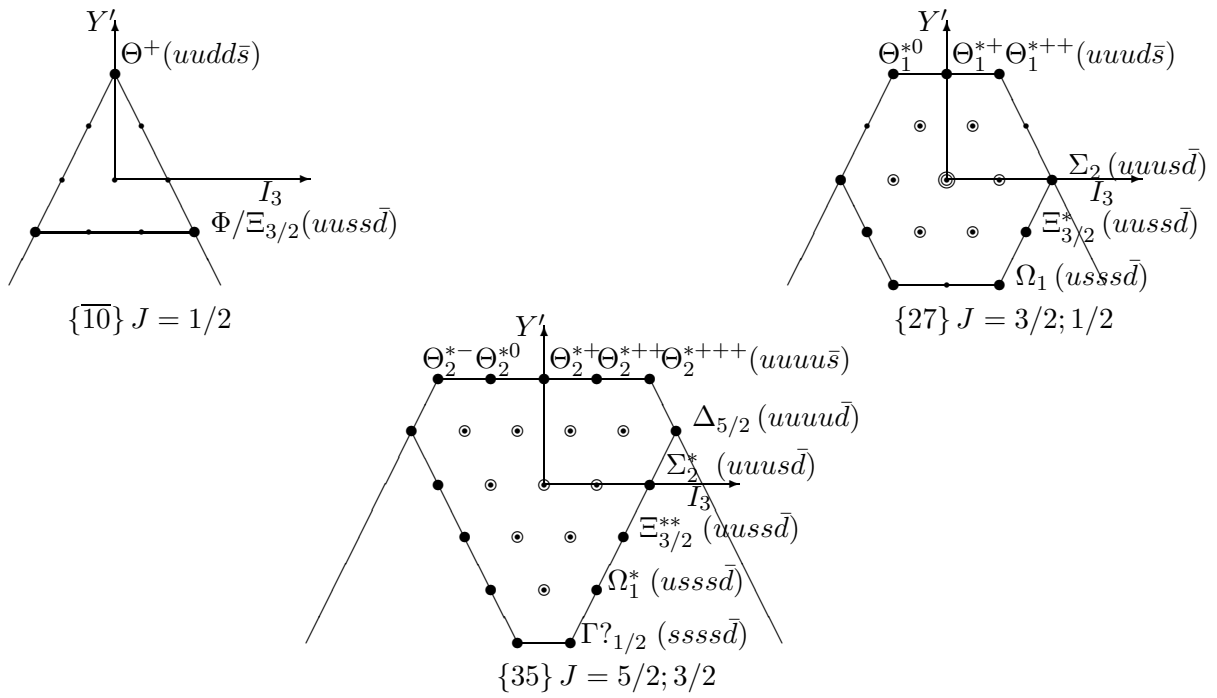


Figure 1: The $I_3 - Y'$ diagrams ($Y' = S + 1$) for multiplets of pentaquark baryons, antidecuplet, $[p, q] = [0, (N_c + 3)/2]$, $\{27\}$ -plet with $[p, q] = [2, (N_c + 1)/2]$ and $\{35\}$ -plet, $[p, q] = [4, (N_c - 1)/2]$. For $N_c > 3$ these diagrams should be extended within long lines, as shown in the picture. Quark contents are given for manifestly exotic states (components with maximal value of I_3), when $N_c = 3$.

$[p, q]; (Y', I)$	$C_S(N)$	$C_S(N = 3)$
$[1, (N - 1)/2]$		
$Y' = 1, I = 1/2$	$2(N + 4)/[(N + 3)(N + 7)]$	7/30
$Y' = 0, I = 0$	$3/(N + 7)$	9/30
$Y' = 0, I = 1$	$(3N + 13)/[(N + 3)(N + 7)]$	11/30
$*Y' = -1, I = 3/2$	$(4N + 18)/[(N + 3)(N + 7)]$	—
$Y' = -1, I = 1/2$	$4/(N + 7)$	12/30
$[3, (N - 3)/2]$		
$Y' = 1, I = 3/2$	$2(N + 4)/[(N + 1)(N + 9)]$	7/24
$Y' = 0, I = 1$	$(3N + 7)/[(N + 1)(N + 9)]$	8/24
$*Y' = -1, I = 5/2$	$(4N + 22)/[(N + 1)(N + 9)]$	—
$Y' = -1, I = 1/2$	$(4N + 6)/[(N + 1)(N + 9)]$	9/24
$*Y' = -2, I = 3$	$(5N + 29)/[(N + 1)(N + 9)]$	—
$Y' = -2, I = 0$	$5/(N + 9)$	10/24
$[0, (N + 3)/2]$		
$Y' = 2, I = 0$	$3/(N + 9)$	6/24
$Y' = 1, I = 1/2$	$(4N + 9)/[(N + 3)(N + 9)]$	7/24
$Y' = 0, I = 1$	$(5N + 9)/[(N + 3)(N + 9)]$	8/24
$Y' = -1, I = 3/2$	$(6N + 9)/[(N + 3)(N + 9)]$	9/24
$*Y' = -2, I = 2$	$(7N + 9)/[(N + 3)(N + 9)]$	—

Table 1. Strangeness contents of the "octet", "decuplet" and "antidecuplet" of baryons at arbitrary $N = N_c$, for unmixed states, $Y' = S + 1$. Few states (marked by *) are shown which appear only if $N > 3$, mostly they are states with maximal possible value of isospin at fixed Y' .

Approximately at large N_c

$$C_S \simeq \frac{2 + |S|}{N_c}. \quad (8)$$

The Gell-Mann — Okubo formula takes place in the form

$$C_S = -A(p, q)Y - B(p, q) \left[Y^2/4 - \vec{I}^2 \right] + C(p, q), \quad (9)$$

$A(p, q), B(p, q), C(p, q)$ depend on particular $SU(3)$ multiplet. For the ‘octet’, for example [7],

$$A(“8”) = \frac{N_c + 2}{(N_c + 3)(N_c + 7)}, \quad B(“8”) = \frac{2}{(N_c + 3)(N_c + 7)}, \quad C(“8”) = \frac{3}{(N_c + 7)}. \quad (10)$$

$[p, q]; (Y', I)$	$C_S(N)$	$C_S(N = 3)$
$[2, (N + 1)/2]$		
$Y' = 2, I = 1$	$(3N + 23)/[(N + 5)(N + 11)]$	32/112
$Y' = 1, I = 3/2$	$(4N^2 + 65N/2 - 3/2)/[(N + 1)(N + 5)(N + 11)]$	33/112
$Y' = 1, I = 1/2$	$(4N + 24)/[(N + 5)(N + 11)]$	36/112
$Y' = 0, I = 2$	$(5N^2 + 39N - 26)/[(N + 1)(N + 5)(N + 11)]$	34/112
$Y' = 0, I = 1$	$(5N^2 + 33N + 8)/[(N + 1)(N + 5)(N + 11)]$	38/112
$Y' = 0, I = 0$	$5/(N + 11)$	5/14
* $Y' = -1, I = 5/2$	$(6N^2 + \frac{91}{2}N - \frac{101}{2})/[(N + 1)(N + 5)(N + 11)]$	—
$Y' = -1, I = 3/2$	$(6N^2 + 38N - 8)/[(N + 1)(N + 5)(N + 11)]$	40/112
$Y' = -1, I = 1/2$	$(6N + 7/2)/[(N + 1)(N + 11)]$	43/112
* $Y' = -2, I = 3$	$(7N^2 + 52N - 75)/[(N + 1)(N + 5)(N + 11)]$	—
$Y' = -2, I = 1$	$(7N + 2)/[(N + 1)(N + 11)]$	46/112
$[4, (N - 1)/2]$		
$Y' = 2, I = 2$	$(3N + 25)/[(N + 3)(N + 13)]$	34/96
$Y' = 1, I = 5/2$	$(4N^2 + 85N/3 - 79)/[(N - 1)(N + 3)(N + 13)]$	21/96
$Y' = 1, I = 3/2$	$(4N + 24)/[(N + 3)(N + 13)]$	36/96
* $Y' = 0, I = 3$	$(5N^2 + \frac{104}{3}N - 133)/[(N - 1)(N + 3)(N + 13)]$	—
$Y' = 0, I = 2$	$(5N^2 + \frac{74}{3}N - 67)/[(N - 1)(N + 3)(N + 13)]$	26/96
$Y' = 0, I = 1$	$(5N + 23)/[(N + 3)(N + 13)]$	38/96
* $Y' = -1, I = 7/2$	$(6N^2 + 41N - 187)/[(N - 1)(N + 3)(N + 13)]$	—
$Y' = -1, I = 3/2$	$(6N^2 + 21N - 55)/[(N - 1)(N + 3)(N + 13)]$	31/96
$Y' = -1, I = 1/2$	$(6N + 22)/[(N + 3)(N + 13)]$	40/96
* $Y' = -2, I = 4$	$(7N^2 + \frac{142}{3}N - 241)/[(N - 1)(N + 3)(N + 13)]$	—
$Y' = -2, I = 1$	$(7N^2 + 52N/3 - 43)/[(N - 1)(N + 3)(N + 13)]$	36/96
$Y' = -2, I = 0$	$7/(N + 13)$	42/96
* $Y' = -3, I = 9/2$	$(8N^2 + \frac{161}{3}N - 295)/[(N - 1)(N + 3)(N + 13)]$	—
$Y' = -3, I = 1/2$	$(8N - 31/3)/[(N - 1)(N + 13)]$	41/96

Table 2. Strangeness contents for unmixed states of the “{27}”-plet (spin $J = 3/2$) and “{35}”-plet ($J = 5/2$) of baryons, for arbitrary $N = N_c$ and numerically for $N_c = 3$. As in Table 1, some states which exist only for $N_c > 3$ (with maximal isospin) are marked with *.

For ‘decuplet’

$$A(“10”) = \frac{N_c + 2}{(N_c + 1)(N_c + 9)}, \quad B(“10”) = \frac{2}{(N_c + 1)(N_c + 9)}, \quad C(“10”) = \frac{3}{(N_c + 9)}, \quad (11)$$

and for ‘antidecuplet’ where for each iso-multiplet a relation takes place $I = (1 - S)/2$ it was possible to obtain the relations

$$A(“\overline{10}”) + \frac{3}{2}B(“\overline{10}”) = \frac{N_c}{(N_c + 3)(N_c + 9)}, \quad C(“10”) - 2B(“\overline{10}”) = \frac{5N_c + 9}{(N_c + 3)(N_c + 9)}. \quad (12)$$

If we try to make expansion in $1/N_c$, then parameter is $\sim 7/N_c$ - for the ‘octet’. For ‘decuplet’ and ‘antidecuplet’ expansion parameter is $\sim 9/N_c$ and becomes worse for greater multiplets, ”{27}”-plet, ”{35}”-plet, etc. Apparently, for realistic world with $N_C = 3$ the $1/N_c$ expansion *does not work*.

Any chain of states connected by relation $I = C' \pm Y/2$ reveals linear dependence on hypercharge (strangeness). Interpretation of these results in terms of strange quark/antiquark masses should be quite careful. For such multiplets as ‘octet’, ‘decuplet’ the CSA *mimics the quark model* with effective strange quark mass

$$m_s^{eff} \sim \tilde{m}_K^2 \Gamma [A(p, q) \mp 3B(p, q)/2]. \quad (13)$$

This is valid if the flavor symmetry breaking (FSB) is included in the lowest order of perturbation theory. At large N_c

$$m_s^{eff} \sim \tilde{m}_K^2 \Gamma / N_c, \quad (14)$$

too much, about $\sim (0.6 - 0.7) GeV$ if extrapolated to $N_c = 3$.

If we make expansion in RRM, we obtain for the ‘octet’ of baryons the contribution to the mass, proportional to \bar{m}_K^2

$$\begin{aligned} \delta M_N &= 2\tilde{m}_K^2 \frac{\Gamma}{N_c} \left(1 - \frac{6}{N_c}\right), & \delta M_\Lambda &= \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 - \frac{21}{N_c}\right), \\ \delta M_\Sigma &= \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 - \frac{17}{N_c}\right), & \delta M_\Xi &= \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(4 - \frac{28}{N_c}\right), \end{aligned} \quad (15)$$

and for decuplet

$$\delta M_\Delta = 2\tilde{m}_K^2 \frac{\Gamma}{N_c} \left(1 - \frac{6}{N_c}\right), \dots \quad \delta M_\Omega = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(5 - \frac{45}{N_c}\right), \quad (15a)$$

equidistantly for all 4 components. Note, that for the ‘nucleon’ and ‘ Δ ’ these contributions to the mass coincide in the leading and next-to-leading orders of $1/N_c$ expansion, and can be considered as contribution of the ‘sea’ of $s\bar{s}$ pairs. The effective strange quark masses estimates and their $1/N_c$ expansion follow from (15) immediately (section 6).

4 Bound state model of skyrmions quantization

Within the bound state model (BSM) [9] anti-kaon or kaon is bound by $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{HFS}, \quad (16)$$

where flavor and antiflavor excitation energies

$$\omega_S = N_c(\mu - 1)/8\Theta_K \simeq \frac{\tilde{m}_K^2 \Gamma}{N_c}, \quad \omega_{\bar{S}} = N_c(\mu + 1)/8\Theta_K \simeq \frac{N_c}{4\Theta_K} + \frac{\tilde{m}_K^2 \Gamma}{N_c}, \quad (17)$$

$$\mu = \sqrt{1 + \tilde{m}_K^2/M_0^2} \simeq 1 + \frac{\tilde{m}_K^2}{2M_0^2} = 1 + 8 \frac{\tilde{m}_K^2 \Gamma \Theta_K}{N_c^2},$$

$$\omega_S + \omega_{\bar{S}} = \frac{\mu N_c}{4\Theta_K} \simeq \frac{N_c}{4\Theta_K} + \frac{\tilde{m}_K^2 N_c}{8\Theta_K M_0^2}.$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_K) \sim N_c^0, \quad \mu \sim N_c^0 \sim 1. \quad (18)$$

The expansion of μ made above really does not work well even for the case of strangeness, however, it is very useful for comparison of BSM and RRM.

The hyperfine splitting correction depending on hyperfine splitting constants c and \bar{c} , isospin I and spin J of the state, and ‘‘strange isospin’’ $I_S = |S|/2$ equals [9]

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \frac{(c_S - 1)[J(J+1) - I(I+1)] + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_\pi}. \quad (19)$$

$$c_S = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu - 1) \simeq 1 - 4\frac{\Theta_\pi\Gamma m_K^2}{N_c^2},$$

$$\bar{c}_S = \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu - 1) \simeq 1 - 8\frac{\Theta_\pi\Gamma m_K^2}{N_c^2}. \quad (20)$$

The approximate equality shown in the right side takes place when expansion in m_K^2 is possible. In this approximation $\bar{c}_S \simeq c_S^2$, as mentioned in the literature. It is a point of principle that in the BSM baryon states are labeled by their strangeness (flavor), spin and isospin, but do not belong apriori to definite $SU(3)$ multiplet (p, q) . They can be a mixture of different $SU(3)$ multiplets, indeed.

For flavor (negative strangeness or beauty, positive charm) the HFS correction disappears if $\bar{m}_K = 0$, and we can rewrite the mass formula for flavored states:

$$M(I, J, S) \simeq M_{cl} + \frac{N_c}{4\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \frac{\bar{m}_K^2\Gamma}{N_c} \left\{ 2 + |S| - \frac{2}{N_c} [J(J+1) - I(I+1) + I_S(I_S+1)] \right\}. \quad (21)$$

It is clear from this expression that the energy is minimal when ‘strange’ isospin is maximal, i.e. $I_S = -S/2$. For decuplet isospin $I = (3+S)/2$ and $I_S(I_S+1) - I(I+1) = -5(3+2S)/4$, therefore, equidistant location of the components of decuplet is reproduced.

In this way we obtain for the ‘octet’ and ‘decuplet’ the contributions depending on m_K^2 :

$$\delta M_N = 2\tilde{m}_K^2 \frac{\Gamma}{N_c}, \quad \delta M_\Lambda = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 - \frac{3}{N_c} \right),$$

$$\delta M_\Sigma = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 + \frac{1}{N_c} \right), \quad \delta M_\Xi = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(4 - \frac{4}{N_c} \right),$$

$$\delta M_\Delta = 2\tilde{m}_K^2 \frac{\Gamma}{N_c} \simeq \delta M_N, \quad \delta M_\Omega = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(5 - \frac{15}{N_c} \right), \quad (22)$$

It is instructive to compare the total splitting of the ‘octet’ and ‘decuplet’ in the BSM and in RRM

$$\Delta_{tot}('8', BSM) = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(2 - \frac{4}{N_c} \right), \quad \Delta_{tot}('8', RRM) = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(2 - \frac{16}{N_c} \right),$$

$$\Delta_{tot}('10', BSM) = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 - \frac{15}{N_c} \right), \quad \dots \quad \Delta_{tot}('10', RRM) = \tilde{m}_K^2 \frac{\Gamma}{N_c} \left(3 - \frac{33}{N_c} \right). \quad (23)$$

In BSM mass splittings are bigger than in RRM.

It follows already from this comparison that the RRM used for prediction of pentaquarks [10] is *different* from the BSM model, used in [11] to disavow the Θ^+ ⁴.

⁴Intensive discussion of the CSA predictions validity for exotic baryon states has been initiated in [12]. However, the explicit difference between RRM and BSM in the next to leading terms of $1/N_c$ expansion of contributions $\sim \tilde{m}_K^2$, being discussed here, was not established in [12].

For antiflavor (positive strangeness or beauty, or negative charm) certain changes should be done: $\omega_S \rightarrow \omega_{\bar{S}}$ and $c_S \rightarrow c_{\bar{S}}$ in the last term. It is crucially important that for the antiflavor the hyperfine splitting constants are different; they can be obtained by means of the change $\mu \rightarrow -\mu$ in above formulas (see, e.g. detailed evaluation in [7]):

$$\begin{aligned} c_{\bar{S}} &= 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu + 1) \simeq 1 - \frac{\Theta_\pi}{\Theta_K} + 4\frac{\Theta_\pi\Gamma m_K^2}{N_c^2} + O(m_K^4), \\ \bar{c}_{\bar{S}} &= 1 + \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu + 1) \simeq 1 + 2\frac{\Theta_\pi}{\Theta_K} - 24\frac{\Theta_\pi\Gamma m_K^2}{N_c^2} + O(m_K^4), \end{aligned} \quad (24)$$

and even approximate inequality of the type $\bar{c}_S \simeq c_S^2$ does not hold for positive strangeness.

As a result, the mass formula for anti-flavored states takes the form:

$$\begin{aligned} M(I, J, S > 0) &\simeq M_{cl} + \frac{N_c + S}{4\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_K} [I(I+1) - J(J+1) + 3I_S(I_S+1)] + \\ &+ \frac{\bar{m}_K^2\Gamma}{N_c} \left\{ 2 + |S| + \frac{2}{N_c} [J(J+1) - I(I+1) - 7I_S(I_S+1)] \right\}. \end{aligned} \quad (25)$$

For antiflavor (positive strangeness, etc.) the term in Eq. (25) $\sim 1/\Theta_K$ is large even for small m_K^2 :

$$\Delta M_{HFS}^{\bar{S}}(\bar{m}_K = 0) = \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_K} \{[-J(J+1) + I(I+1)] + 3I_S(I_S+1)\}. \quad (18a)$$

This contribution to the position of the baryon mass agrees with the result of the RR model.

The case of exotic $S = +1$ Θ hyperons is especially interesting. For $\Theta_0^+ \in \overline{10}$, $J = 1/2$, $I = 0$, and we obtain

$$M_{\Theta_0, J=1/2} = M_{cl} + \frac{2N_c + 3}{4\Theta_K} + \frac{3}{8\Theta_\pi} + \bar{m}_K^2\Gamma \left(\frac{3}{N_c} - \frac{9}{N_c^2} \right),$$

For $\Theta_1^+ \in \{27\}$, $J = 3/2$, $I = 1$, and we have

$$M_{\Theta_1, J=3/2} = M_{cl} + \frac{2N_c + 1}{4\Theta_K} + \frac{15}{8\Theta_\pi} + \bar{m}_K^2\Gamma \left(\frac{3}{N_c} - \frac{7}{N_c^2} \right),$$

For $\Theta_0^+ \in \{35\}$ $J = 5/2$, $I = 2$, and the contribution to the mass is

$$M_{\Theta_2, J=5/2} = M_{cl} + \frac{2N_c - 1}{4\Theta_K} + \frac{35}{8\Theta_\pi} + \bar{m}_K^2\Gamma \left(\frac{3}{N_c} - \frac{5}{N_c^2} \right). \quad (26)$$

The terms $\sim 1/\Theta_K$ agree with those obtained in RRM for anti-decuplet, $\{27\}$ - and $\{35\}$ -plets (terms, proportional to $K(p, q, J)$ in the RRM mass formula). This means that, indeed, we can interpret these positive strangeness states as belonging to definite $SU(3)$ multiplets — antidecuplet, $\{27\}$ - and $\{35\}$ -plets⁵, at least when expansion of the quantity μ , we made above. is possible.

We should compare also the contributions $\sim \bar{m}_K^2\Gamma$ with the mass splitting correction from RRM:

$$\delta M_{\Theta_0, J=1/2}^{RRM} = \bar{m}_K^2\Gamma \left(\frac{3}{N_c} - \frac{27}{N_c^2} \right), \quad \delta M_{\Theta_1, J=3/2}^{RRM} = \bar{m}_K^2\Gamma \left(\frac{3}{N_c} - \frac{25}{N_c^2} \right),$$

⁵It is an unresolved problem, however, how to obtain other components of these multiplets within the BSM. Evaluations performed in the literature are not sufficient for this purpose. The point is that, for example, strange isospin which is unique for the states with strangeness $S = \pm 1$, is uncertain for the components of exotic multiplets different from the $S = 1$ states [7].

$$\delta M_{\Theta_2, J=5/2}^{RRM} = \bar{m}_K^2 \Gamma \left(\frac{3}{N_c} - \frac{23}{N_c^2} \right), \quad (27)$$

and again — as in case of ‘octet’ and ‘decuplet’ — considerable difference takes place between the RRM and BSM results.

The addition of the term to the BSM result, *possible due to normal ordering ambiguity for the operators of (anti)strangeness production, present in BSM* (I.Klebanov, VBK, 2005, unpublished)

$$\Delta M_{BSM-RRM} = -6\bar{m}_K^2 \frac{\Gamma}{N_c^2} (2 + |S|) \quad (28)$$

brings results of RRM and BSM in agreement — for nonexotic as well as exotic $S = +1$ states. This procedure looks however not quite satisfactorily: if we believe to RRM, why we need BSM at all? Anyway, RRM and BSM in its accepted form are *different models*.

The rotation-vibration approach (RVA) by H.Weigel and H.Walliser [13] tries to unify RRM and BSM in some way, Θ^+ has been confirmed with somewhat higher energy and considerable width ($\Gamma_\Theta \sim 50 \text{ MeV}$)⁶.

5 The role of configuration mixing

Configuration mixing due to the term $\sim m_K^2 \Gamma s_\nu^2$ in the lagrangian [14] is important, e.g. the Δ state from decuplet of baryons is mixed with the Δ' state from $\{27\}$ -plet, and as a result, the splitting between these states becomes larger: the mass of the Δ goes down, and the mass of Δ' increases, see Fig.2. Similar mixing takes place for other baryon states which have equal values of strangeness and isospin but belong to different $SU(3)$ multiplets.

For anti-decuplet mixing decreases slightly the total splitting, and pushes the N^* and Σ^* states toward higher energy. Mixing with components of the octet is important. Apparent contradiction with the simplest assumption of equality of masses of strange quarks and antiquarks $m(s) = m(\bar{s})$ takes place (see next section).

For decuplet mixing increases total splitting considerably, but approximate **equidistancy still remains!**⁷. Mixing with the components of $\{27\}$ -plet is important, e.g. the $\Delta \in \{10\}$ after mixing with $\Delta^* \in \{27\}$ moves to the lower value of mass.

A note for the QM should be made: states with different numbers of $q\bar{q}$ pairs can mix, and such mixing *should be taken into account*. In the diquark-diquark-antiquark picture proposed in [16] the mixing of pentaquark states with the ground state baryon octet should be included, since strong interactions do not conserve the number of quark-antiquark pairs present in the hadron. This mixing pushes the pentaquark states towards higher energy and changes the whole picture of relative positions of baryon states. Without such mixing the diquark picture [16] looks artificial, whereas within CSA this problem is resolved in natural way.

We conclude this section with the following discussion of the case of large value of the mass \bar{m}_F , which, besides m_K , can be also \bar{m}_D or \bar{m}_B . When this mass is large enough, the expansion of the quantity μ in (17) cannot be made, and instead of this expansion we have

⁶The alternative RRM — BSM is not resolved appropriately in the literature. In some cases when there is an ambiguity, the priority is given to the RR model (see, e.g. [13], sections 3 and 4). The hyperfine splitting correction in [13] has the form different from our. According to Eq. (3.21) of [13] it is

$$\Delta M_S = \frac{1}{2\Theta_\pi} + [c_S J(J+1) + (1 - c_S)I(I+1) + c_S(c_S - 1)/4], \quad (3.21)$$

the last term being completely different from our in Eq. (19). In view of this, the authors of [13] stated: "The comparison with the RRA suggests that these quartic terms contribute $9/8\Theta_K$ to the mass of the $S = 1$ baryons." According to our BSM formulas we have $(\bar{c}_S - c_S)I_S(I_S + 1)/(2\Theta_\pi)|_{\bar{m}_K^2=0} = 9/(8\Theta_K)$ in agreement with the RRM, and there is no need to correct the BSM formulas ‘by hands’.

⁷Therefore, statement made in several papers that approximate equidistancy within the decuplet of baryons is an argument that configuration mixing is negligible, is not correct

$\mu \simeq \bar{m}_F/M_0 = 4\bar{m}_F\sqrt{\Gamma\Theta_K}/N_c$. This linear dependence of μ and also flavor excitation energies $\omega_F, \bar{\omega}_F$ on the mass m_F , given by (17), is quite reasonable, but

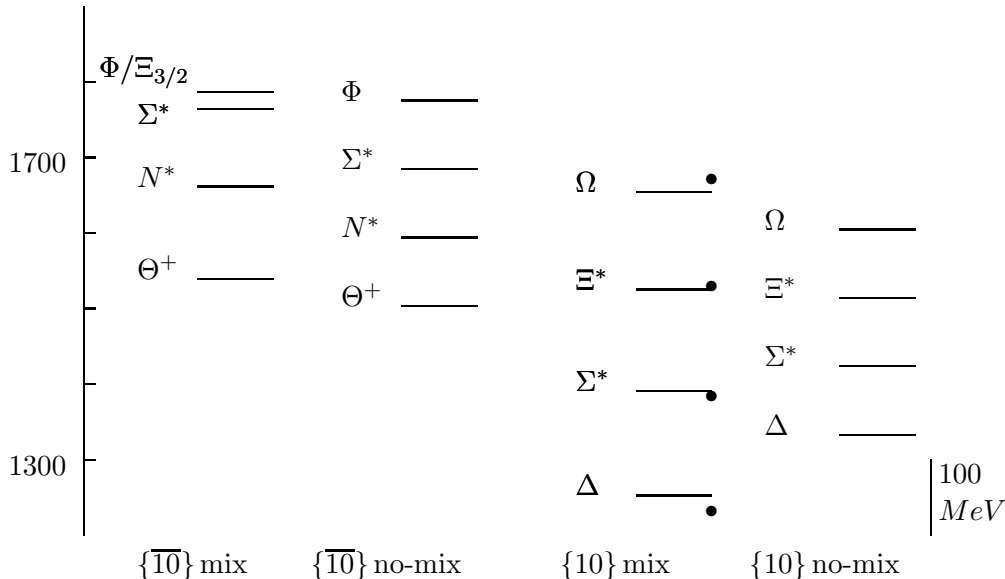


Figure 2: *Influence of the configuration mixing* (H.Yabu, K.Ando, 1988 [14]) on the mass splitting within antidecuplet and decuplet of baryons, RR model (variant by H.Walliser [15]). For decuplet the data are shown by black points.

it is not possible to ascribe the quantized states uniquely to definite $SU(3)$ irreps, as we made above in section 4. It is a challenging problem to get such linear in m_F behaviour of flavored states energies within rotator models, RRM or SRM. Probably, strong configuration mixing which should take place in this case, would be able to reduce the quadratic dependence on m_F (or linear in Γ) and to convert it to linear dependence. Numerical calculations with configuration mixing program arranged by H.Walliser and used in [15] confirm this point, but an analytical proof is desirable.

6 Comparison of CSA results with simple quark model

It is possible to make comparison of the CSA results with expectations from simple quark model in *pentaquark* approximation (projection of CSM on QM). The masses $m_s, m_{\bar{s}}$ and the mass of $s\bar{s}$ pair $m(s\bar{s})$ come into play, as presented in Table 3 for pure states (without mixing). Examples of wave functions of pentaquarks (PQ-s) in the diquark-diquark-antiquark picture by R.Jaffe and F.Wilczek [16] are the following (see also [18], [6].):

$$\Theta_0 \in \{\bar{10}\} \sim [ud][ud]\bar{s},$$

where $[ud]$ is diquark with zero isospin (singlet in flavor). Other states can be obtained, e.g. by action of the operator U^- which transforms d -quark into s -quark, and $\bar{s} \rightarrow \bar{d}$, $U^-\bar{s} = -\bar{d}$, and well known isotopic I^\pm operators. For example,

$$N^{*+} \in \{\bar{10}\} \sim [\sqrt{2}\bar{s}\{[us][ud]\} - \bar{d}[ud][ud]]/\sqrt{3}, \dots$$

$$\Phi/\Xi_{3/2}^- \in \{\bar{10}\} \sim [sd][sd]\bar{u}, \dots \Phi/\Xi_{3/2}^+ \in \{\bar{10}\} \sim [su][su]\bar{d}.$$

For larger N_c the number of diquarks, equal to $N_D = (N_c + 1)/2$, increases, and additional $s\bar{s}$ pairs appear in wave functions of some states ⁸.

⁸Standard assumption is that the baryon number of the quark equals to $1/N_c$. We accept also the relation between hypercharge and strangeness in the form $Y = S + N_c B/3$ (see, e.g. [12]). Note that the quantity

For antidecuplet at arbitrary N_c according to Fig.1 and Table 1 any state with strangeness S has isospin $I = (1 - S)/2$ and its mass equals

$$M(\overline{10}, S, I = (1 - S)/2) = M(\overline{10}, S = 1, I = 0) + \bar{m}_K^2 \Gamma \frac{(1 - S)N_c}{(N_c + 3)(N_c + 9)}, \quad (29)$$

Interpretation of this relation in terms of the quark model is not straightforward. Simple relations can be obtained from Table 3 for the effective s -quark and antiquark masses m_s and $m_{\bar{s}}$, from the total splitting of antidecuplet ($N_c = 3$)

$$[2m_s - m_{\bar{s}}]_{\overline{10}} = \bar{m}_K^2 \frac{\Gamma}{8}, \quad (30)$$

which equals numerically 272 MeV for the parameters accepted in [15]. For arbitrary number of colors this relation should be rewritten as

$$[(N_c + 1)m_s - 2m_{\bar{s}}]_{\overline{10}} = \bar{m}_K^2 \frac{N_c \Gamma}{N_c + 9}. \quad (30a)$$

Configuration mixing decreases this quantity to 247 MeV , see Table 3. Relation (30) is the only relation which can be obtained, according to Table 3. If we assume that the strange quark mass in antidecuplet is the same as in decuplet, $m_s(\overline{10}) = m_s(10)$, then the mass of strange antiquark should be negative if the configuration mixing is not included: $m_{\bar{s}}(\overline{10}) = -m_s(\overline{10})$. This relation looks unrealistic. Note, that if the mass of s -antiquark within antidecuplet were equal to that of s -quark (we call this variant *simplistic* model), then this splitting would be much smaller, just equal to $m_s \sim 130 \text{ MeV}$. A natural way out of this contradiction is to allow the masses of strange quark/antiquark within antidecuplet to be different from those within decuplet and other multiplets.

It is remarkable that configuration mixing pushes the splitting towards simplistic quark model where the splitting of antidecuplet should be about m_s , since $m_{\bar{s}} \simeq m_s$. If we assume that the s -quark mass in $\overline{10}$ is about 150 MeV , as in the decuplet, then strange antiquark within $\overline{10}$ should be very light, with the mass about $30 - 50 \text{ MeV}$.

$ \overline{10}, 2, 0 \rangle$	$ \overline{10}, 1, \frac{1}{2} \rangle$	$ \overline{10}, 0, 1 \rangle$	$ \overline{10}, -1, \frac{3}{2} \rangle$		
$m_{\bar{s}}$	$2m_{s\bar{s}}/3$	$m_s + m_{s\bar{s}}/3$	$2m_s$		
564	655	745	836		
600	722	825	847		
$ 27, 2, 1 \rangle$	$ 27, 1, \frac{3}{2} \rangle$	$ 27, 0, 2 \rangle$	$ 27, -1, \frac{3}{2} \rangle$	$ 27, -2, 1 \rangle$	
$m_{\bar{s}}$	$m_{s\bar{s}}/2$	m_s	$2m_s$	$3m_s$	
733	753	772	889	1005	
749	887	779	911	1048	
$ 35, 2, 2 \rangle$	$ 35, 1, \frac{5}{2} \rangle$	$ 35, 0, 2 \rangle$	$ 35, -1, \frac{3}{2} \rangle$	$ 35, -2, 1 \rangle$	$ 35, -3, \frac{1}{2} \rangle$
$m_{\bar{s}}$	0	m_s	$2m_s$	$3m_s$	$4m_s$
1152	857	971	1084	1197	1311
1122	853	979	1107	1236	1367

Table 3. Strange quark (antiquark) masses contributions (for the case $N_c = 3$) and calculation results within RRM, without and with configuration mixing (1-st and 2-d lines of numbers, correspondingly [15]). For each value of strangeness the states with largest value of isospin are considered here.

Y' in Fig.1 and Tables 1,2 is by definition $Y' = S + 1$. The wave function of 'pentaquark' in this case is " $\Theta_0 \in \overline{10} \sim [ud] \dots [ud] \bar{s}$, with $(N_c + 1)/2$ $[ud]$ diquarks, etc.

For the components of $\{27\}$ -plet with strangeness $S \leq -1$ the relation takes place

$$M(27, S, I = (5 + S)/2) = M(27, S = -1, I = 2) - \bar{m}_K^2 \Gamma \frac{(S + 1)(N_c^2 - N_c + 18)}{(N_c + 1)(N_c + 5)(N_c + 11)}, \quad (31)$$

and for $N_c = 3$ we get $m_s^{eff}(27) \simeq 3\bar{m}_K^2 \Gamma / 56 \simeq 117 \text{ MeV}$. which increases to 135 MeV when configuration mixing is included.

From splittings within the $\{27\}$ -plet between components with $Y' \geq 0$ we get also

$$[m_s - m_{\bar{s}}]_{27} = \bar{m}_K^2 \Gamma / 56,$$

which equals numerically to 39 MeV [15] and reduces to 30 MeV when configuration mixing is included.

It is of interest that when configuration mixing is not included then the mass of strange quark-antiquark pair $m_{s\bar{s}} = (m_s + m_{\bar{s}})/2$ both for antidecuplet and $\{27\}$ -plet. This relation is the consequence of Gell-Mann — Okubo relation, indeed. For arbitrary N_c interpretation of formula (31) in terms of effective quark/antiquark masses becomes more difficult, since additional $s\bar{s}$ pairs are present in simple wave functions.

Let us consider now the biggest (in multiplicity) pentaquark. The remarkable property of $\{35\}$ -plet is that the lowest in mass state is not the state with highest value of hypercharge, $Y' = 2$, but the state in the middle of the multiplet, which has $Y' = 1$, $S = 0$, $I = 5/2$. In the pentaquark approximation ($N_c = 3$) this state contains neither strange quark/antiquark, no $s\bar{s}$ pair, and has the smallest numerically strangeness content among all baryons considered here. As can be seen from Table 3, the mass of $s\bar{s}$ pair does not enter the masses of all $\{35\}$ -plet components with largest values of isospin. The masses of these states with $S \leq 0$ are connected by relation

$$M(35, S, I = 5/2 + S/2) = M(35, S = 0, I = 5/2) - \bar{m}_K^2 \Gamma \frac{S(N_c^2 + 12 - 11N_c/3)}{(N_c - 1)(N_c + 3)(N_c + 13)}, \quad (32)$$

so, for $N_c = 3$ the quantity

$$m_s^{eff}(35) = \bar{m}_K^2 \Gamma \frac{5}{96} \simeq 114 \text{ MeV} \quad (33)$$

can be considered as an effective strange quark mass in this case. Configuration mixing increases this quantity to 130 MeV (see Table 3).

From the difference between masses of $S = +1$ and $S = 0$ states we can extract the effective strange antiquark mass

$$[m_{\bar{s}}]_{35} = \bar{m}_K^2 \Gamma \frac{13}{96} \simeq 295 \text{ MeV}, \quad (35)$$

which is remarkably large value. Configuration mixing reduces slightly this quantity to 270 MeV .

For arbitrary N_c we can get an idea about the behavior of strange antiquark mass for antidecuplet, $\{27\}$ -plet and $\{35\}$ -plet if we make some assumption about the contribution of strange quark sea, in particular, that it is the same as for ‘nucleon’ and ‘ Δ ’-isobar (coinciding in the leading and next-to-leading orders of $1/N_c$ expansion, see Table 1).

In this way we obtain from the RRM (the contribution of the sea of $s\bar{s}$ pairs is subtracted):

$$[m_{\bar{s}}]_{10} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{15}{N_c}\right); \quad [m_{\bar{s}}]_{27} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{13}{N_c}\right); \quad [m_{\bar{s}}]_{35} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{11}{N_c}\right), \quad (36)$$

and within the BSM

$$[m_{\bar{s}}]_{10} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{9}{N_c}\right); \quad [m_{\bar{s}}]_{27} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{7}{N_c}\right); \quad [m_{\bar{s}}]_{35} \sim \frac{\bar{m}_K^2 \Gamma}{N_c} \left(1 - \frac{5}{N_c}\right). \quad (37)$$

So, we see from here that numerical results shown in **Table 3** can be understood qualitatively by this expansion, although extrapolation back to the real $N_c = 3$ world is not possible. It is worth noting also that the changes of the effective s -antiquark mass from antidecuplet to $\{35\}$ -plet are equal within RRM and BSM, although the mass itself is smaller in RRM — in the next to leading order of $1/N_c$ expansion.

We summarize our results for the first two terms of the $1/N_c$ expansion of the effective strange quark and antiquark masses in Table 4. ‘Octet’ and ‘decuplet’ of baryons do not contain valent $s\bar{s}$ pairs, the mass difference between components is defined entirely by valent strange quarks. The mass m_s is defined as a half of the total splitting for the ‘octet’ and $1/3$ of the total splitting for ‘decuplet’.

	$\{8\}$	$\{10\}$	$\{\overline{10}\}$	$\{27\}$	$\{35\}$
m_s^{RRM}	$1 - 8/N_c$	$1 - 11/N_c$	—	—	—
m_s^{BSM}	$1 - 2/N_c$	$1 - 5/N_c$	—	—	—
$m_{\bar{s}}^{RRM}$	—	—	$1 - 15/N_c$	$1 - 13/N_c$	$1 - 11/N_c$
$m_{\bar{s}}^{BSM}$	—	—	$1 - 9/N_c$	$1 - 7/N_c$	$1 - 5/N_c$

Table 4. First terms of the $1/N_c$ expansion for the effective strange quark and antiquark masses within different $SU(3)$ multiplets, in units $\bar{m}_K^2 \Gamma / N_c$. Empty spaces are left in the cases of theoretical uncertainty. The assumption concerning strange quarks/antiquarks sea, described in the text, should be kept in mind.

Strong dependence of the s -antiquark mass on the multiplet is required when we project the results of CSA on simple quark model: is it artefact of CSA, or is physically significant — is not clear now. The influence of the configuration mixing on the contribution of m_s , $m_{\bar{s}}$, $m_{s\bar{s}}$ to baryon states should be included in more detailed consideration.

7 Diquarks mass difference estimates

Estimates of the diquark mass differences can be made roughly using results obtained from CSA. As it was suggested by F.Wilczek [17] the singlet in spin diquark $[q_1 q_2]$, which is antitriplet $\bar{3}_F$ in flavor, is called ‘good’ diquark d_0 , the triplet in spin diquark $(q_1 q_2)$, which is 6_F in flavor, is called ‘bad’ diquark d_1 . Both ‘good’ and ‘bad’ diquarks are antitriplets in color. As it was presented in previous section, the wave function for pentaquarks from antidecuplet can be written in terms of diquark wave functions [16], [18] as:

$$\Theta_0 \in \{\overline{10}\} \sim [ud][ud]\bar{s}, \dots \Phi/\Xi_{3/2}^- \in \{\overline{10}\} \sim [sd][sd]\bar{u} \dots$$

It is not possible to built $\{27\}$ and $\{35\}$ -plets from ‘good’ diquarks only, ‘bad’ diquarks are needed, as can be illustrated well by these examples of wave functions of positive strangeness baryons:

$$\Theta_1^0 \in \{27\} \sim (dd)[ud]\bar{s}, \Theta_1^+ \in \{27\} \sim (ud)[ud]\bar{s}, \Theta_1^{++} \in \{27\} \sim (uu)[ud]\bar{s},$$

$$\Theta_2^- \in \{35\} \sim (dd)(dd)\bar{s}, \Theta_2^0 \in \{35\} \sim (ud)(dd)\bar{s}, \dots \Theta_2^{+++} \in \{35\} \sim (uu)(uu)\bar{s}.$$

It seems to be natural to ascribe the difference of rotation energies for different multiplets, given by the term $\sim K(p, q, I_R)$ in expression (5) to the difference of masses of ‘bad’ and ‘good’ diquarks. Since ‘bad’ diquark is heavier, this is obvious reason why Θ_1 is heavier than Θ_0 , and Θ_2 is more heavy.

From the difference of $\{27\}$ -plet and antidecuplet masses

$$M(d_1) - M(d_0) \sim \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 100 \text{ MeV}. \quad (38)$$

from $\{35\}$ -plet and $\{27\}$ -plet mass difference

$$M(d_1) - M(d_0) \sim \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 250 \text{ MeV}. \quad (39)$$

Qualitatively this result seems to be OK, in agreement with previous estimates [17] and, e.g., lattice calculations [19], but this picture should be too naive. In particular, interaction between diquarks may be important, which makes the $\Theta_{5/2}$ even more heavy.

8 Rigid Rotator — Soft Rotator dilemma

The rigid rotator model is a limiting case of the rotator model when deformations of skyrmions during rotation in $SU(3)$ configuration space are totally neglected. In the soft rotator model, opposite to rigid rotator, it is supposed that soliton is deformed under influence of FSB forces: static energy minimization is made at fixed value of ν . Dependence on ν of static characteristics of skyrmions is taken into account in the quantization procedure.

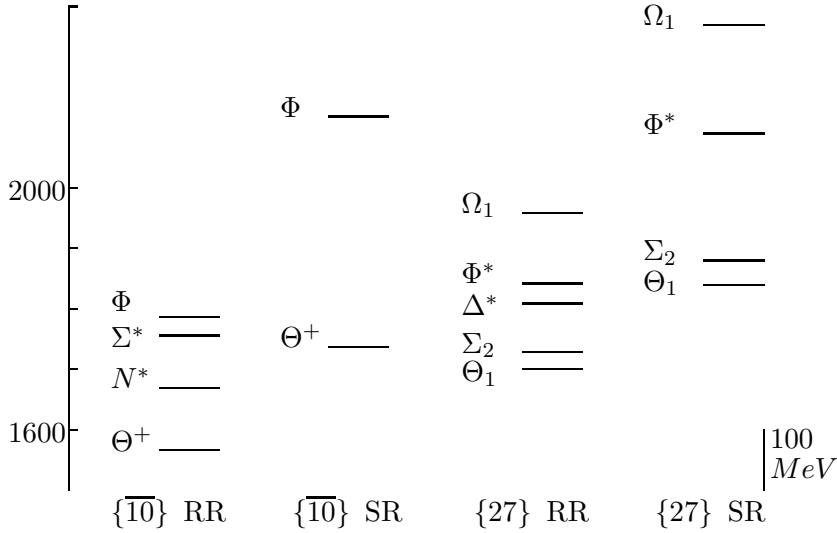


Figure 3: Comparison of the rigid rotator (RR) and soft rotator (SR) models predictions for the masses of exotic baryons, antidecuplet and $\{27\}$ -plets. Not all states are shown for the SR model. The code for SR model used here was arranged by B.Schwesinger, H.Weigel (1992)[20].

Static characteristics of skyrmions depend on ν - the angle of rotation into ‘strange’ direction. For ‘strange’, or kaonic inertia it is most important:

$$\Theta_K = \frac{1}{8} \int (1 - c_f) \left[F_K^2 - \sin^2 \nu (F_K^2 - F_\pi^2) (2 - c_f) / 2 + \frac{1}{e^2} \left(f'^2 + \frac{2s_f^2}{r^2} \right) \right] d^3 r. \quad (40)$$

It is decreasing function of $\sin^2 \nu$. RRM corresponds to $\nu = 0$, the maximal value of kaonic inertia Θ_K and relatively low values of masses of exotic baryons (like Θ , $\Phi/\Xi_{3/2}$, etc.). Within SRM the masses of baryons from antidecuplet and $\{27\}$ - plet are considerably greater than in RRM, mostly due to smaller value of Θ_K (see Fig.3). The truth is somewhere between RR and SR models, but to make reasonable calculation seems to be unrealistic presently since the properties of baryonic matter are not known, in particular the response of matter relative to the FSB forces.

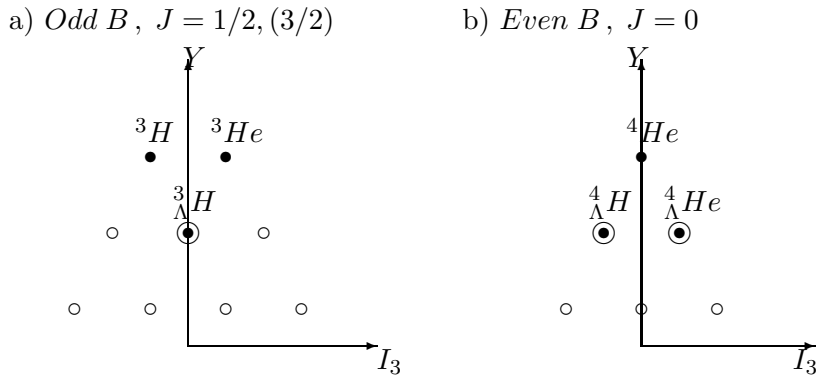


Figure 4: (a) The location of the isoscalar state (shown by double circle) with odd B -number and $|S| = 1$ in the upper part of the $(I_3 - Y)$ diagram. (b) The same for isodoublet states with even B . The case of light hypernuclei ${}_{\Lambda}H$ and ${}_{\Lambda}He$ is presented as an example. The lower parts of diagrams with $Y \leq B - 3$ are not shown here.

9 Strange multibaryons or hypernuclei

Great advantage of CSA is that multibaryon states — nuclei, hypernuclei ... — can be considered on equal footing with the $B = 1$ case. The rational map approximation proposed in [21] simplifies this work considerably and allows to calculate easily all static characteristics of multiskyrmions necessary for spectrum evaluation. In particular, the B -number dependence of quantities of interest has been established, $\Theta_I \sim B$, $\Theta_J \sim B^2$ for $B \leq 20 - 30$. Some kind of the “bag model” for multibaryons can be obtained with the help of this ansatz, starting with effective lagrangian [22].

Ordinary nuclei and hypernuclei (ground states) can be ascribed to definite $SU(3)$ multiplets, as shown in Fig.4 for baryon numbers 3 and 4. In a version of BSM it is possible to describe total binding energies of light hypernuclei in qualitative, even semiquantitative agreement with data [23]. The collective motion of multiskyrmion in the $SU(3)$ collective coordinates space is taken into account. The results of such estimates within the rigid oscillator model (a variant of the bound state model) are presented in Fig.5, and quite satisfactory qualitative agreement with existing data on total binding energies takes place.

For the $B = 2$ case more detailed investigations have been performed. The lowest multiplets of dibaryons are shown in Fig.6: left figure shows the antidecuplet of the $J = 1$ dibaryons, the $I = 0$ deuteron being the nonstrange state; right figure shows the $J = 0$ {27}-plet, the $I = 1$ nucleon-nucleon scattering state being the upper (nonstrange) component. There is also {35}-plet with the $N\Delta$ -like nonstrange upper component (isospin $I = 2$) and {28} - plet with the $\Delta\Delta$ -like upper component (isospin $I = 3$). {28} - plet contains the state with $S = -6$ (di-Omega). {35}-plet and {28} - plet are not shown in Fig.6.

Calculations of spectrum of strange dibaryons have been performed in [24] in SRM which is more relevant for the $B = 2$ case than for the $B = 1$ case. When the NN -scattering state was fitted to be on the right place (the deuteron binding energy is then about 30 Mev), all strange and multistrange dibaryons are above threshold by few tens of Mev, so, they can appear as near-threshold enhancements in scattering cross sections of baryons with appropriate quantum numbers. These results are in qualitative agreement with quark models calculations [25].

Multibaryons with positive strangeness or beauty (or negative charm) have been predicted within similar approach as well [26].

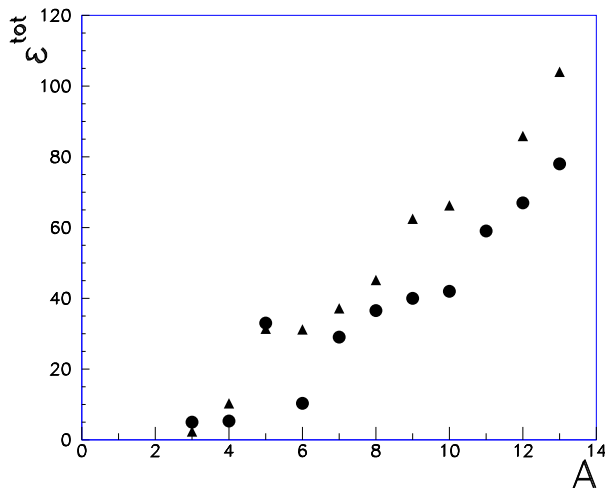


Figure 5: Total binding energies of light hypernuclei (in MeV). Full triangles — experimental data, full circles — theoretical results in a version of BSM applied to multiskyrmions [23]. Figure from [4]b.

Rotational excitations of any state have additional energy

$$\Delta E = \frac{J(J+1)}{2\Theta_J}. \quad (41)$$

$J = 2^+$ excited states have energy by $\sim 2/\Theta_J$ greater than $\bar{10}$. The state with $S = -1$, $I = 1$, $J^P = 2^+$ can be interpreted as $NN\bar{K}$ state with binding energy ~ 100 Mev. For the $B = 2$ {27}-plet $J = 1$ states have energy by $1/\Theta_J \sim 60$ Mev greater than $J = 0$ ground states.

The orbital inertia grows fast with increasing baryon (atomic) number, $\Theta_J \sim B^p$, where p is between 1 and 2. By this reason the number of rotational states becomes larger for large baryon numbers. Some of them can be interpreted as *deeply bound anti-kaon states* discussed intensively in [27] and other papers. More detailed investigations of this issue are necessary.

10 Summary and conclusions

We can summarize our discussion in the following way:

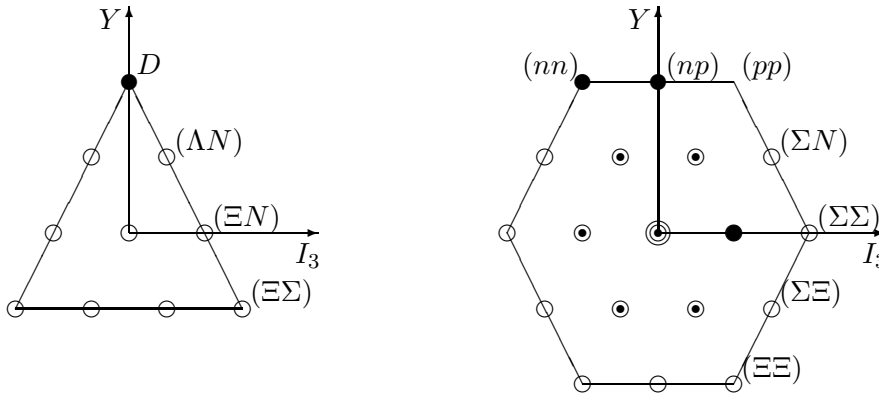
The parameter of $1/N_c$ expansion is large for the case of the baryon spectrum, extrapolation to real world is not possible in this way, and conclusions made in the limit $N_c \rightarrow \infty$ may not be valid in the real world. Rigid (soft as well) rotator and bound state models coincide in the first order of $1/N_c$ expansion, but *differ* in the next orders.

Configuration mixing is important, according to RRM, and makes substantial influence on the effective quark masses within simple quark model.

Transition to Soft Rotator Model from RRM may be crucial, leading to the increase of masses, especially for exotic states.

There is correspondence of the chiral soliton RRM and quark model predictions for pentaquarks spectra in negative S sector of {27} and {35} plets: the effective mass of strange quark is about $135 - 130$ MeV, slightly smaller for {35}.

For positive strangeness components the link between CSM and QM requires strong dependence of effective \bar{s} mass on particular $SU(3)$ multiplet. The $1/N_c$ expansion for the effective strange antiquark mass provides different results within rotator and bound state models in the next-to-leading order, but the changes of the effective $m_{\bar{s}}$ when we go from one multiplet to another are equal for the RRM and BSM. Configuration mixing pushes spectra towards *simplistic*



$$B = 2, \{\overline{10}\}, J = 1 (2, \dots)$$

$$B = 2, \{27\}, J = 0 (1, \dots)$$

Figure 6: $I_3 - Y$ diagrams of multiplets of dibaryons, $B = 2$: the $J = 1$ antidecuplet (should not be mixed up with antidecuplet of pentaquarks, $B = 1$) and the $J = 0$ $\{27\}$ -plet. Virtual levels (scattering states) are shown in brackets, e.g. (ΛN) scattering state which appears as near-threshold enhancement.

model - nice property, but reasons for this are not clear presently. Diquarks mass difference estimates from CSA seem to be reasonable.

As conclusion, we state that chiral soliton models, based on few principles and ingredients incorporated in effective lagrangian, allow to describe qualitatively, in some cases even quantitatively, various characteristics of baryons and nuclei — from ordinary ($S = 0$) nuclei to known hypernuclei. This suggests that predictions of pentaquark states, as well as multibaryons with strangeness, are of interest. Existence of PQ by itself is without any doubt, although very narrow PQ may not exist. Wide, even very wide PQ should exist, and searches for PQ-s remain to be an actual task.

There are, however, problems when one tries to project results of the CSA on quark models: strong dependence of strange antiquark mass on the $SU(3)$ multiplet; difference of masses of ‘bad’ and ‘good’ diquarks is not unique in naive picture, at least.

In view of theoretical uncertainties, experimental investigations are of crucial importance. In particular, experiments at the J-PARC accelerator (50 GeV) can provide great chance to shed more light on the puzzles of baryon spectroscopy.

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