

## Visual Basic Programs for Spreadsheet Analysis

by Bruce Hunt<sup>1</sup>

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### Abstract

A collection of Visual Basic programs, entitled Function.xls, has been written for ground water spreadsheet calculations. This collection includes programs for calculating mathematical functions and for evaluating analytical solutions in ground water hydraulics and contaminant transport. Several spreadsheet examples are given to illustrate their use.

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### Introduction

Numerous people have used spreadsheets for analyzing pumping test data. For example, Johnson et al. (2001) used a spreadsheet as an interface for a finite-difference model when analyzing well test data, and Jiao and Leung (2003) gave a review and critique of spreadsheet software that has been made available by the U.S. Geological Survey (Halford and Kuniansky 2002) for the analysis of aquifer-test and slug-test data. Commercial spreadsheet packages, such as AQTESOLV and AquiferTest, are also available. This computer note describes, and illustrates the use of, a collection of user-defined functions that have been written in Visual Basic for Applications for use in Excel spreadsheets for the analysis of ground water problems. This collection of programs is entitled Function.xls.

### Description of Function

All programs are contained in four attached modules, which have been entitled MathFunctions, Hydraulics, Dispersion, and ScaleDepDispersion. The MathFunctions module contains programs for computing Bessel functions, error functions with both positive and negative arguments, exponential integrals, the complete and incomplete gamma function, the leaky aquifer function and its derivatives, the complete and incomplete beta function, the Airy function, definite integrals using both the trapezoidal and Simpson's

one-third rule, and the Stehfest algorithm for the numerical inversion of the Laplace transform.

The Hydraulics module contains ground water hydraulics programs based on analytical solutions. These solutions include the Theis solution for flow to a well, the Jacob-Hantush solution for flow to a well in a leaky aquifer, the Boulton solution for flow to a well in a delayed-yield aquifer, the Hunt (2003) solution for flow to a well in a delayed-yield aquifer beside a stream, the Hunt (2004) solution for flow depleted from a spring by a well in a delayed-yield aquifer, the rise in ground water levels from aquifer recharge over rectangular and circular areas, and unsteady drawdowns and pore velocities created by horizontal, inclined, and partially penetrating vertical wells in leaky aquifers.

The Dispersion module contains programs for calculating contaminant dispersion from analytical solutions for various types of sources in uniform flow. These solutions, which assume constant dispersion coefficients, include both instantaneous and continuous sources in one, two, and three dimensions. The ScaleDepDispersion module contains programs for dispersion solutions obtained by Hunt (1998, 2002) when dispersivities increase with the first power of distance downstream from the point of release.

### Example Applications

The first example, shown in Figure 1, illustrates the use of the Boulton solution program for fitting a calculated curve to measured drawdown data. Parameters and variables that are held constant during the calculations are entered in the first two rows of the spreadsheet. The parameters  $a$ ,  $b$ , and  $n$ , which are also entered in the first two rows, have been used to compute  $n$  points from  $10^a$  to  $10^b$

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	A	B	C	D	E	F	G	H	I									
1	Q(m <sup>3</sup> /min)	r (m)	T (m <sup>2</sup> /min)	S	K'/B' (min <sup>-1</sup> )	σ	a	b	n									
2	1.2	25	1.5	0.0013	0.0006	0.065	-1	3	50									
3																		
4	The Boulton solution for flow to a well																	
5																		
6	Measured Values			Calculated Values														
7	t (min)	s (m)		k	t (min)	s (m)												
8	0.5	0.056		1	0.1	0.0076754												
9	1	0.082		2	0.1206793	0.0109918												
10	2	0.103		3	0.1456348	0.0150439												
11	3	0.111		4	0.1757511	0.0198094												
12	5	0.117		5	0.2120951	0.0252346												
13	10	0.121		6	0.2559548	0.0312408												
14	15	0.124		7	0.3088844	0.0377303												
15																		
16																		
17																		
18																		
19																		
20																		
21																		
22																		
23																		
24																		
25																		
26																		
27																		
28																		
29																		
30																		
31	1000	0.288		24	7.5481261	0.1157167												
32	1200	0.292		25	9.1029818	0.1206929												
33				26	10.985411	0.1216823												
34				27	13.257114	0.1227778												

Figure 1. A spreadsheet example for fitting the Boulton solution to field data.

that are equally spaced on a log<sub>10</sub> scale by using the formula:

$$t_k = 10^{M_k} \quad (1)$$

where the exponent  $M_k$  is given by

$$M_k = a + (k - 1)(b - a)/(n - 1) \quad (2)$$

Values of  $k$  have been entered in column D, values of  $t$  have been calculated with Equation 1 in column E and corresponding values of drawdown have been calculated with the spreadsheet program in column F using an equation of the form:

$$= (Q/T) * W_3(r * \text{SQRT}(KB/T), E8 * KB/S, S/\sigma, 1) \quad (3)$$

where  $Q$ ,  $T$ ,  $r$ ,  $KB$ ,  $S$ , and  $\sigma$  represent the contents of cells \$A\$2, \$C\$2, \$B\$2, \$E\$2, \$D\$2, and \$F\$2, respectively; E8 refers to the relevant time in column E; and  $W_3$  is the macro function provided in the Hydraulics module of Function.xls. Measured drawdowns have been entered in columns A and B and then superimposed as filled circles upon the plot of calculated drawdowns.

One of the great advantages of spreadsheet calculations is that any change in value for a parameter is automatically incorporated into the plot. This makes it a very straightforward matter to vary aquifer parameters to obtain a good fit to data provided that the effect of each parameter upon the calculated curve is known. The Boulton solution contains four parameters—the transmissivity,  $T$ , and elastic storage coefficient,  $S$ , for the pumped aquifer; the ratio of permeability to thickness,  $K'/B'$ , for the overlying aquitard; and the effective porosity (specific yield),  $\sigma$ , at the location of the free surface in the overlying aquitard. A general description of the Boulton solution behavior is that the first portion of the drawdown curve, a-b in Figure 1, occurs before leakage becomes important, and, therefore, is approximated closely by the Theis solution for a confined aquifer. Thus, the horizontal position of segment a-b is determined by  $S$ , and a decrease in  $S$  translates a-b leftward without changing the shape of the segment. Furthermore, the asymptotic expression used for the Jacob straight-line approximation shows that  $T$  controls the slope of the straight-line asymptote to a-b and that increasing  $T$  causes a decrease in the asymptote slope. (In some problems, such as the one shown in Figure 1, segment a-b is not sufficiently long to allow this asymptote to be completely reached.) The

horizontal segment c-d extends over the period of time when downward leakage from the overlying aquitard nearly balances flow abstracted from the well. Therefore, the vertical position of this segment is controlled by changing  $K'/B'$ , with a decrease in this parameter moving segment c-d in the upward direction. For example, decreasing  $K'/B'$  in Figure 1 would move the segment upward and, at the same time, allow segment a-b to come closer to its straight-line asymptote. Finally, segment d-e occurs when free-surface drawdowns in the aquitard approach drawdowns in the pumped aquifer. Then downward leakage through the aquitard gradually decreases to zero, and draw-down responses in both the aquitard and pumped aquifer become identical and are approximated with the solution for an unconfined aquifer with an effective porosity at the free surface of  $\sigma$ . Thus,  $T$  controls the slope of the straight-line asymptote for segment d-e, and changes in  $\sigma$  cause d-e to translate horizontally without changing the shape of the segment. More specifically, an increase in  $T$  decreases the asymptote slope, and a decrease in  $\sigma$  causes d-e to move leftward.

The leftward translation of segments a-b and d-e, caused by decreasing  $S$  and  $\sigma$ , respectively, is explained by noting that a decrease in either of these two parameters decreases the amount of water that can be released from storage. The end result is that continuity requires draw-

downs to increase, and this appears in the plot as a leftward translation of the segment. One consequence of this is that horizontal spacing between the straight-line asymptotes of segments a-b and d-e is fixed by the ratio of  $S/\sigma$ . The upward translation of segment c-d, caused by decreasing  $K'/B'$ , occurs because a decrease in value of this parameter decreases the rate at which aquifer recharge from leakage occurs, and this creates a corresponding increase in draw-down.

There are at least two good reasons for using semi-log plots rather than arithmetic or log-log plots when analyzing data from a pumping test. First, the asymptotic formula used for the Jacob straight-line approximation shows that asymptotes for segments a-b and d-e appear as straight lines in a semi-log plot. Secondly, semi-log plots avoid weighting the fit between measured and simulated drawdowns toward small drawdowns, as occurs with log-log plots.

There are, of course, other ways in which programs in Function.xls can be used to obtain aquifer parameters when comparing measured and calculated solutions. Optimization routines such as Solver or least squares can be used, or graphical aids such as slide bars can be placed beside the plots. However, these devices are easier to incorporate into more-specialized spreadsheets that use one particular solution to analyze field data. Examples of this more specialized use of Function.xls, which include spreadsheet

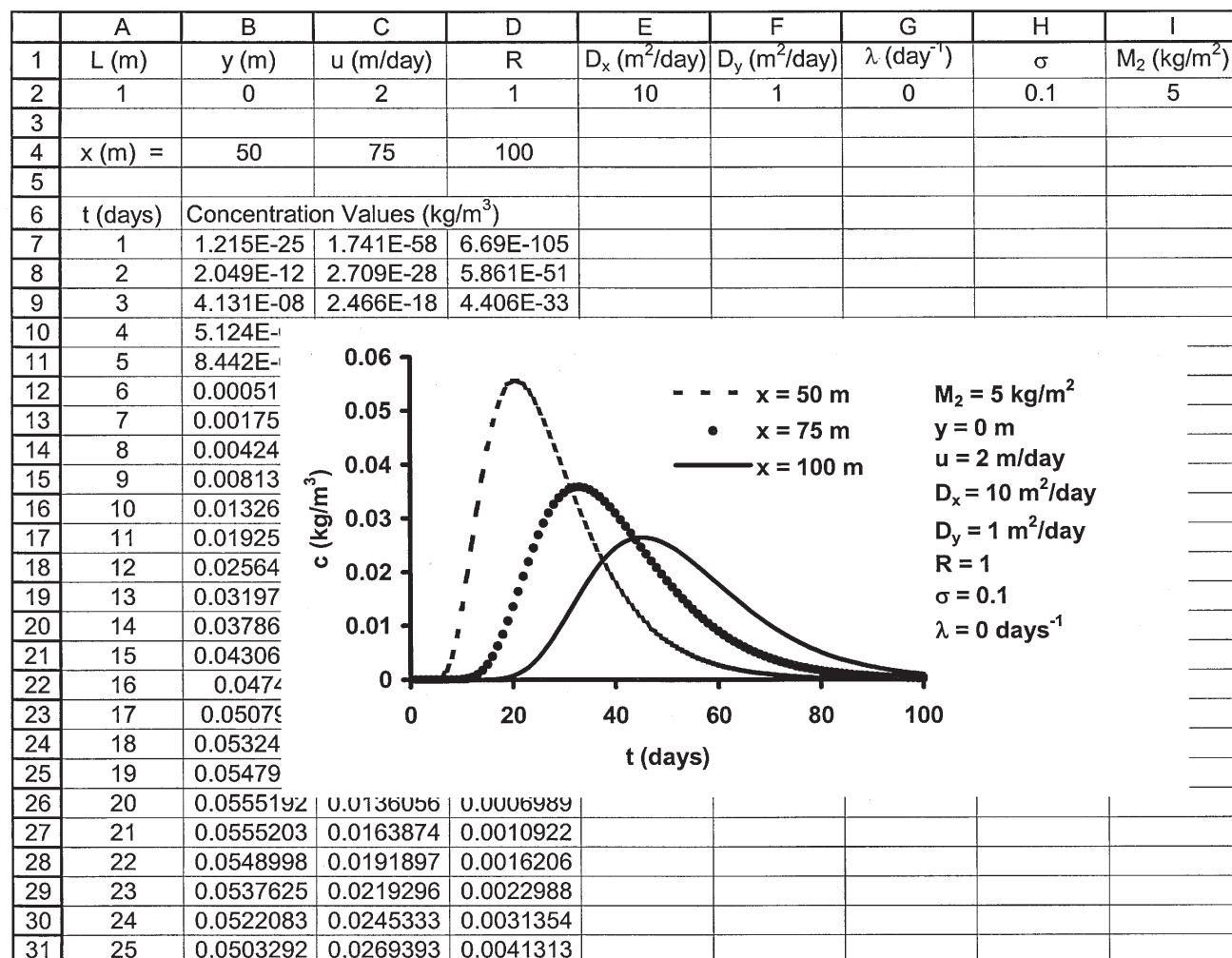


Figure 2. A spreadsheet example for the dispersion of a contaminant from a two-dimensional instantaneous point source.

solutions for the Theis, Jacob-Hantush, and stream-depletion solutions, have been prepared by David Scott of Environment Canterbury and are available without charge at the Web site <http://www.ecan.govt.nz/gwapps.html>. On the other hand, the method of manual adjustment described herein is easily understood and implemented, and is reasonably efficient. It also helps lead to a better understanding of the problem physics, which makes it an ideal tool for student instruction.

A second example, shown in Figure 2, illustrates the use of Function.xls to calculate concentrations for the dispersion of a contaminant from a two-dimensional instantaneous point source. This is a solution of the following problem:

$$D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} = u \frac{\partial c}{\partial x} + R \frac{\partial c}{\partial t} + R\lambda c \quad (-\infty < x < \infty, -\infty < y < \infty, 0 < t < \infty) \quad (4)$$

$$c(x,y,0) = \frac{M_2}{\sigma} \delta(x) \delta(y) \quad (-\infty < x < \infty, -\infty < y < \infty) \quad (5)$$

$$\lim_{r \rightarrow \infty} c(x,y,t) = 0 \quad (r \equiv \sqrt{x^2 + y^2}, 0 < t < \infty) \quad (6)$$

where  $c$  = concentration in units of mass per unit volume of water;  $D_x$ ,  $D_y$  = dispersion coefficients in the  $x$  and  $y$  directions, respectively;  $t$  = time;  $u$  = pore velocity in the  $x$  direction (the specific discharge divided by the aquifer porosity,  $\sigma$ );  $R$  = retardation factor (dimensionless);  $\lambda$  = decay constant; and  $M_2$  = injected mass of the contaminant divided by the aquifer thickness. Dirac's delta function is denoted by  $\delta(x)$ . Concentration histories have been plotted in Figure 2 for three different values of  $x$  along the streamline  $y = 0$ , which is the streamline where the contaminant was injected instantaneously at  $t = 0$ . Function.xls also contains programs to evaluate continuous source solutions in one, two, and three dimensions, all of which have been obtained by analytically distributing instantaneous sources over time. Concentrations for pulses and other types of time-varying sources can be calculated numerically in a spreadsheet by superimposing solutions for continuous sources.

## Conclusions

A collection of Visual Basic programs for use in spreadsheet applications, entitled Function.xls, has been described and applied to two examples. The first example illustrated how the program for the Boulton solution can be used with field data to obtain aquifer parameters; the second example calculated contaminant concentrations for dispersion from a two-dimensional instantaneous point source. All programs in Function are easily implemented and are suitable for use in more specialized software packages.

## Software Availability

The software Function.xls, together with a manual that gives detailed examples of its use, may be downloaded without charge from the Web site <http://www.civil.canterbury.ac.nz/staff/bhunt.asp>.

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