Cosmic equation of state, Gravitational lensing Statistics and Merging of Galaxies

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Abstract

In this paper we investigate observational constraints on the cosmic equation of state of dark energy ($p = w\rho$) using gravitational lensing statistics. We carry out likelihood analysis of the lens surveys to constrain the cosmological parameters Ω_m and w. We start by constraining Ω_m and w in the no-evolution model of galaxies where the comoving number density of galaxies is constant. We extend our study to evolutionary models of galaxies - Volmerange & Guiderdoni Model and Fast-Merging Model (of Broadhurst, Ellis & Glazebrook). For the no-evolution model we get $w \leq -0.24$ and $\Omega_m \leq 0.48$ at 1σ (68% confidence level). For the Volmerange & Guiderdoni Model we have $w \leq -0.2$ and $\Omega_m \leq 0.58$ at 1σ, and for the Fast Merging Model we get $w \leq -0.02$ and $\Omega_m \leq 0.93$ at 1σ. For the case of constant Λ

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($w = -1$), all the models permit $\Omega_m = 0.3$ with 68% CL. We observe that the constraints on w and Ω_m (and on Ω_m in the case of $w = -1$) obtained in the case of evolutionary models are weaker than those obtained in the case of the no-evolution model.

1 Introduction

Recent observations are in concordance with flat comological models in which the universe is in an accelerating phase. Analysis of the magnitude-redshift data of high-redshift type Ia supernovae (SNe Ia) suggests that the ratio of the matter energy to the critical energy, Ω_m, is ~ 0.3.[1–3] These calibrated "standard" candles appear fainter than would be expected if the expansion was slowing due to gravity. Recent studies of the cosmic microwave background (CMB) data also favour a nearly flat universe: the first acoustic peak in the angular power spectrum of the CMB is located at $l \sim (197 \pm 6)$.[4] Theoretical modelling of structure formation based on cold dark matter models (CDM) with $\Omega_m = 1$ fails to reconcile with the observations at the quantitative level. By contrast, a spatially flat ΛCDM universe (non-zero cosmological constant) with $\Omega_m \simeq 0.3$ explains observations of galaxy clustering on large scales, increases the age of the universe (which helps to accommodate the age of globular clusters), and makes the total energy density equal to critical density as generically predicted by inflationary cosmology.[5,6]

Neither observational evidence, nor inflationary considerations tell us that the cosmological term (dominant, negative pressure term) is a constant. There are various phenomenological models of dynamical-Λ (dark energy component) present in the literature. These are:

1. Λ varies with cosmic time or with the scale factor of Friedmann-Robertson-Walker (FRW) metric.[7–10]

2. An X-matter cosmology where this unknown form of energy is characterised by an equation of state $p_x = w\rho_x$.[11–14]

3. Rolling scalar field models (quintessence): The Λ term is considered to be a new, physical, classical field with some phenomenological Lagrangian. In this class the most popular models are those of an evolving scalar field that couples minimally to gravity.[15–17]

In the present work, we focus our attention on this dark energy component characterised by the equation of state, namely the ratio $w = p/\rho$. In particular, we work with an equation of state with $-1 < w < 0$ because this range fits current cosmological observations best. The cosmological constant is a special case corresponding to $w = -1$. Constraints from large scale structure (LSS) and CMB complemented by the SNe Ia data require $0.6 \leq \Omega_x \leq 0.7$ and $w < -0.6$ for a flat universe[2,18] and $w < -0.4$ for universes with arbitrary spatial curvature.[18] Alcaniz and Lima (1999)[19] derive the limits on the cosmic equation of state from age measurements of old high redshift galaxies (OHRG). They show that if, as indicated from dynamical measurements, the density parameter $\Omega_m \sim 0.3$, then $w \leq -0.27$. By combining "cosmic concordance" with maximum likelihood estimator, Wang *et al.* (2000)[20] find that the best-fit model lies in the range $\Omega_m = 0.33 \pm 0.05$ with an effective equation of state $w \sim -0.65 \pm 0.07$.

We use the statistics of gravitational lensing of quasars to set quantitative limits on the present density of dark energy component and matter.

The main aim of this paper is to costrain the cosmic equation of state using gravitational lensing statistics as a tool, taking into account the *evolution of lenses (galaxies).* Many observations suggest that galaxies we see today could have evolved from the merging of smaller subsystems. Hence inclusion of the fact that the number density of lensing galaxies changes with time is very important in the lensing statistics.[21–23] We consider two different evolutionary models of galaxies : the fast merger model of Broadhurst, Ellis & Glazebrook (1992)[24] and the Volmerange & Guiderdoni model.[25] Both the models consider the number evolution of galaxies in addition to the pure luminosity evolution.

The paper is organized as follows. In Section 2, we review the "dark energy component" model. In section 3, discuss the two evolutionary models we are considering. In section 4, we write down the lensing probabilities for the three models: non-evolutionary model (where the comoving number density of lensing galaxies remains constant), fast-merging and Volmerange & Guiderdoni model. We also discuss likelihood analysis of the lens surveys. Section 5 is devoted to a discussion of the results.

2 Field Equations and Distance Formula

For spatially flat, homogeneous, and isotropic cosmologies with non-relativistic matter and a separately conserved "dark energy component" with equation of state $p = w\rho$, the Einstein field equations are:

$$(\frac{\dot{a}}{a})^2 = H_o^2\,[\Omega_m(\frac{a_o}{a})^3 + \Omega_x(\frac{a_o}{a})^{3(1+w)}] \tag{1}$$

and

$$\frac{\ddot{a}}{a} = -\frac{H_o^2}{2}\,[\Omega_m(\frac{a_o}{a})^3 + (3w+1)\Omega_x(\frac{a_o}{a})^{3(1+w)}], \tag{2}$$

where the overdot denotes derivatives with respect to time, H is the Hubble parameter and a is the scale factor. Subscript o refers to present values. Ω_m and Ω_x are present day matter and dark energy density parameters. As we are mostly interested in effects that occured at redshift $z < 5$, we neglect radiation. Hence the flat condition constraint becomes $\Omega_x = 1 - \Omega_m$.

The proper distance between the source at redshift z and the observer at $z = 0$ is

$$d(0,z) = a_o\chi(z) = c\,H_o^{-1}\int_0^z \frac{dy}{\sqrt{\Omega_m(y)^3 + (1-\Omega_m)(y)^{3(1+w)}}}, \tag{3}$$

where $\chi(z)$ is the comoving distance to the source.

The angular diameter distance for our models, between redshift z_L and z_S reads:

$$d_A(z_L, z_S) = \frac{c\,H_o^{-1}}{1+z_S}\int_{z_L}^{z_S} \frac{dy}{\sqrt{\Omega_m(y)^3 + (1-\Omega_m)(y)^{3(1+w)}}}. \tag{4}$$

3 Evolution Of Galaxies

Gravitational lens statistics and galaxy evolution are linked with each other because the galaxies which are evolving through different mechanisms are basically acting as lenses. Therefore the gravitational lens statistics is certainly going to be affected if the lenses (galaxies) evolve. The theory of the formation and evolution of galaxies is one of the major unsolved problems of astrophysics. According to one view, galaxies evolved through a complex series of interactions and have settled in the present day form.[26,27] Others believe that galaxies were created in a well defined event at a very early time in the history of the universe.[28,29] It remains unclear which process dominates the formation of elliptical galaxies. Among the many theories of formation, the idea that galaxies may form by the accumulation of smaller star-forming subsystems has recently received much attention.

Observational evidence also supports this "bottom - up" scheme. Deep Hubble Space Telescope (HST) images[30] and the size-redshift relation of luminous early-type galaxies (E/S0) and mid-type spiral galaxies (Sabc) indicate that these objects were assembled largely before $z = 1$ and have been evolving passively for $z < 1$. Moreover, HST and ground-based telescopes show that the galaxy-merger rate was higher in the past and it roughly increases with redshift.[31–33] These arguments suggest that the galaxies we see today could have been assembled from the merging of smaller systems sometime during $z > 1$.

Recent observations[34] show that there is a strong deficit of galaxies with extreme red colours (seen in different separated fields and at different flux limits) than predicted by models in which elliptical galaxies completed their star formation by $z \sim 5$. Therefore, elliptical galaxies must have had significant star formation at $z < 5$ through merging and associated star bursts. The formation of elliptical galaxies in this way is also consistent with the predictions of hierarchical clustering models of galaxy formation.

The other piece of evidence which supports the merging hypothesis comes from the excess of Faint Blue Galaxies [FBG] which have been found in many deep imaging studies.[35–38] Comparison with a model which assumes no luminosity evolution in the galaxy population shows that, at B = 24, the actual galaxy count exceeds the model predictions by a factor of 5. Merging of galaxies can also solve the surprisingly steep increase in the number density of galaxies.[24,25,39] It is also found that the "FBG problem" cannot be resolved in the conventional scenario of formation and evolution of ellipticals. In this scenario, elliptical galaxies are assumed to have formed in an instantaneous burst of star formation at high redshifts with no subsequent star formation events.

Although galaxy mergers are no longer a matter of dispute, there is no agreement on the current and past rate of galaxy mergers. Several authors have attempted to find the dependence of merger rate on the redshift.[40,41] There are many challenges for the mergers theory which may be met in the near future with more observations. Recently there have been several semi-analytic models, motivated by cosmological theory which may eventually provide the greatest understanding of galaxy formation and evolution.[42–45] The traditional galaxy number-counts models, however, are still powerful tools in exploring the formation and evolution of galaxies and can be treated as complements to the semi-analytic techniques. To explain the galaxy number-counts various number-luminosity evolution models have been constructed. Along with the no-evolution model, we consider two different models of galaxy evolution which try to explain some of the observational facts listed above. These models are:

1. Rocca-Volmerange and Guiderdoni model.[25]
2. Fast merging model.[24]

The general philosophy behind these merger models is to assume that the current giant galaxies result from the merging of a number of smaller "building blocks". It is found that under some general assumptions the theoretical predictions of merging models nicely fit the observed galaxy counts. The assumptions underlying these models are:

1. Stability of the Schechter form for the luminosity function.
2. Conservation of total comoving mass density.

The evolving luminosity function at any redshift z is given as

$$\Phi(L,z)\,dL = \phi_*(z)\ \left(\frac{L}{L_*(z)}\right)^{\alpha}\ exp\left(-\frac{L}{L_*(z)}\right)\ \frac{dL}{L_*(z)} \qquad (5)$$

$L_*(z)$ being the characteristic luminosity at the knee, $\phi_*(z)$ a characteristic density and α is the index of faint end slope.

No-Evolution Model

This model assumes that the comoving number density of galaxies is constant and the mass of galaxies does not change with cosmic time. Therefore

$$\phi_*(z) = \phi_*(0) = \text{constant} \tag{6}$$

The characteristic luminosity at any redshift remains constant and hence the mass of galaxy at any redshift is constant.

$$L_*(z) = L_*(0) = \text{constant} \tag{7}$$

The "0" refers to present-day values.
This is the most commonly used luminosity function of lens galaxies. Therefore eq.(5) becomes

$$\Phi(L,z)\, dL \;=\; \Phi(L,z=0)dL \;=\; \phi_* \left(\frac{L}{L_*}\right)^{\alpha} \; exp\left(-\frac{L}{L_*}\right) \frac{dL}{L_*} \tag{8}$$

where ϕ_*, α and L_* are the normalization factor, the index of faint-end slope and the characteristic luminosity at the present epoch respectively. These values are fixed in order to fit the current luminosities and densities of galaxies. This is known as the Schechter form of the luminosity function.[46]

Volmerange and Guiderdoni Model

In 1990, Volmerange and Guiderdoni,[25] proposed a unifying model to explain faint galaxy counts as well as observational properties of distant radio galaxies. This model of galaxy evolution (hereafter VG model) is based on number evolution in addition to pure luminosity evolution. According to this model the present day galaxies result from the merging of a large number of building blocks and the comoving number of these building blocks evolves as $(1+z)^{1.5}$.

It is argued that the present luminosity function is the well known Schechter Luminosity Function[46] given in eq.(8) above. Then at high z, the galaxies follow a New Luminosity Function where $\phi_*(z)$ and $L_*(z)$ vary as:

$$\phi_*(z) \;=\; \phi_*(0)\,(1+z)^{\eta} \tag{9}$$

and

$$L_*(z) \;=\; L_*(0)\,(1+z)^{-\eta} \tag{10}$$

Now eq.(5) becomes

$$\Phi(L,z)\,dL = (1+z)^{2\eta}\,\phi_*\;\left[\frac{L}{L_*}(1+z)^{\eta}\right]^{\alpha}\;exp\left[\frac{-L}{L_*}(1+z)^{\eta}\right]\frac{dL}{L_*} \tag{11}$$

or

$$\Phi(L,z)\,dL \;=\; (1+z)^{2\eta}\;\Phi(L(1+z)^{\eta}, z=0)\,dL. \tag{12}$$

It is seen that the value $\eta = 1.5$ gives a fair fit to the data on high redshift galaxies. The functional form has the following properties:

(i) Self-similarity as suggested by the Press-Schechter formalism subject to the constraint that the total mass of associated material is conserved.[47]

(ii) The comoving number density evolves as $\phi_*(z) \;=\; \phi_*(0)\,(1+z)^{\eta}$ and the characteristic luminosity of the galaxy luminosity function vary as $L_*(z) \;=\; L_*(0)\,(1+z)^{-\eta}$. Thus the galaxies are more numerous and less massive for increasing z.

Fast Merging Model

The fast merger model of Broadhurst, Ellis & Glazebrook (1992) was originally motivated by the faint galaxy population counts.[24] This model also assumes the comoving number density of the lenses to be a function of the look back time δt or redshift z and hence:

$$\phi_*(\delta t) = f(\delta t)\ \phi_*(0) \tag{13}$$

and

$$L_*(\delta t) = [f(\delta t)]^{-1}\, L_*(0). \tag{14}$$

Since luminosity is related to the velocity dispersion of the dark halo of the lensing galaxy through the Faber-Jackson relation ($L \propto v^\gamma$),[32] this form implies that if we had n galaxies at time δt each with velocity dispersion v, they would by today have merged into one galaxy with a velocity dispersion $[f(\delta t)]^{\frac{1}{\gamma}} v$. The strength and the time dependence of merging is described by the function $f(\delta t)$:

$$f(\delta t) = exp\,(Q\, H_0\, \delta t) \tag{15}$$

where H_0 is the Hubble constant at the present epoch and Q represents the merging rate. We take Q = 4.[24] The look back time δt is related to the redshift z through

$$H_0 \delta t = \int_0^z \frac{(1+y)^{-1}\, dy}{\sqrt{\Omega_m (1+y)^3 + (1-\Omega_m)(1+y)^{3(1+w)}}}. \qquad (16)$$

Now we can rewrite eq.(5) in terms of δt instead of z.

$$\Phi(L, \delta t)\, dL = f(\delta t)^2\, \phi_* \ \left[\frac{L}{L_*} f(\delta t)\right]^\alpha \ exp\left[\frac{-L}{L_*} f(\delta t)\right] \frac{dL}{L_*} \qquad (17)$$

or

$$\Phi(L, \delta t)\, dL = f(\delta t)^2 \ \left[\ \Phi(\ L\, f(\delta t),\ z = 0\ )\ \right] dL \qquad (18)$$

4 Likelihood Analysis of Lens Surveys

4.1 *Basic Equations of Gravitational Lensing Statistics*

For simplicity we use the Singular Isothermal Model (SIS) for a lens mass distribution. The cross-section for lensing events for the SIS model is given by[50]

$$\sigma = 16\, \pi^3\, v^4 (\frac{D_{OL} D_{LS}}{D_{OS}})^2, \qquad (19)$$

where v is the velocity dispersion of the dark halo of a lensing galaxy, D_{OL} is the angular diameter distance to the lens, D_{OS} is the angular diameter distance to the source and D_{LS} is the angular diameter distance between the lens and the source. The mean image separation for the lens at z_L takes a simple form

$$\Delta\Theta = 8\, \pi\, (\frac{v}{c})^2\, \frac{D_{LS}}{D_{OS}}. \qquad (20)$$

The differential probability $d\tau$ of a beam having a lensing event in traversing dz_L is

$$d\tau = n_L(z)\,\sigma\,\frac{cdt}{dz_L}dz_L, \tag{21}$$

where $n_L(z)$ is comoving number density of the lenses and the quantity cdt/dz_L is calculated (in the models we are working on) to be

$$\frac{cdt}{dz_L} = \frac{a_o}{(1+z_L)}\,\frac{1}{\sqrt{\Omega_m(1+z_L)^3 + (1-\Omega_m)(1+z_L)^{3(1+w)}}}. \tag{22}$$

Subsituting for σ from eq. (19), we get

$$d\tau = n_L(z)\left[\frac{16\pi^3}{cH_0^3}v^4(\frac{D_{OL}D_{LS}}{a_o D_{OS}})^2\frac{1}{a_o}\right]\frac{cdt}{dz_L}dz_L. \tag{23}$$

The No-Evolution Model

The luminosity function is given by eq. (8) and therefore we have

$$< n_L(z)\,v^4 >= (1+z_L)^3\,v_*^4\int_0^\infty \Phi(L)dL\left(\frac{v}{v_*}\right)^4. \tag{24}$$

We assume that v is linked with L through the Faber-Jackson relation for elliptical galaxies

$$\left(\frac{L}{L_*}\right) = \left(\frac{v}{v_*}\right)^\gamma. \tag{25}$$

Hence eq.(24) becomes

$$< n_L(z)\,v^4 >= (1+z_L)^3\,v_*^4\int_0^\infty \Phi(L)dL\left(\frac{L}{L_*}\right)^{\frac{4}{\gamma}}. \tag{26}$$

The optical depth can be written as

$$d\tau = F^*(1+z_L)^3 \left(\frac{D_{OL}D_{LS}}{a_o D_{OS}}\right)^2 \frac{1}{a_o}\frac{cdt}{dz_L}dz_L, \qquad (27)$$

with

$$F^* = \frac{16\pi^3}{cH_0^3}\phi_* v_*^4 \Gamma\left(\alpha + \frac{4}{\gamma} + 1\right) \approx 0.026 \qquad (28)$$

for $\phi_* = 1.4 \pm 0.17 h^3\, 10^{-2} Mpc^{-3}$, $\alpha = -1.0$,[49] $\gamma = 4$ and $v_* = 225\, km/s$.[32] F^* is a dimensionless quantity which governs the probability of a beam encountering a lensing object. It is a measure of the effectiveness of matter in producing multiple images.[50]

The differential optical depth of lensing in traversing dz_L with angular separation between ϕ and $\phi + d\phi$, in the presence of evolution of galaxies, is given by

$$\frac{d^2\tau}{dz_L d\phi}d\phi dz_L = F^*\,(1+z_L)^3\,(\frac{D_{OL}D_{LS}}{a_o D_{OS}})^2\,\frac{1}{a_o}\,\frac{cdt}{dz_L}\,\frac{\gamma/2}{\Gamma(\alpha+1+\frac{4}{\gamma})} (\frac{D_{OS}}{D_{LS}}\phi)^{\frac{\gamma}{2}(\alpha+1+\frac{4}{\gamma})}\, exp[-(\frac{D_{OS}}{D_{LS}}\phi)^{\frac{\gamma}{2}}]\frac{d\phi}{\phi}dz_L \qquad (29)$$

for ellipticals (lenticulars), where $\phi = \Delta\Theta/8\pi(v^\star/c)^2$ with $v^\star$ the velocity dispersion corresponding to the characteristic luminosity $L^\star$ in (25).

The Evolutionary Models

In the case where the comoving number density of the lenses changes with time, the equations (26), (27) and (29) read as[23]

$$< n_L(z)\, v^4 >= (1+z_L)^3\, v_*^4 \int_0^\infty \Phi(L,z) dL \left(\frac{L}{L_*}\right)^{\frac{4}{\gamma}}, \qquad (30)$$

$$d\tau = F^*(1+z_L)^3 \left(\frac{D_{OL}D_{LS}}{a_0 D_{OS}}\right)^2 \Psi(\delta t)^{(1-\frac{4}{\gamma})} \frac{1}{a_0} \frac{cdt}{dz_L} dz_L, \qquad (31)$$

where $\Psi(\delta t) = f(\delta t)$ for the fast merging model and $\Psi(\delta t) = (1+z)^{1.5}$ for the VG model for evolution of lensing galaxies.
And

$$\begin{aligned} \frac{d^2\tau}{dz_L d\phi} d\phi dz_L &= F^*\,(1+z_L)^3\,(\frac{D_{OL}D_{LS}}{a_o D_{OS}})^2 \,\frac{1}{a_o}\,\frac{cdt}{dz_L} \\ & \Psi(\delta t)^{2+\alpha}\,\frac{\gamma/2}{\Gamma(\alpha+1+\frac{4}{\gamma})}\,(\frac{D_{OS}}{D_{LS}}\phi)^{\frac{\gamma}{2}(\alpha+1+\frac{4}{\gamma})} \\ & exp[-(\frac{D_{OS}}{D_{LS}}\phi)^{\frac{\gamma}{2}}\,\Psi(\delta t)]\,\frac{d\phi}{\phi}\,dz_L \qquad (32) \end{aligned}$$

At $\gamma = 4$ the differential probability is the same for both evolutionary model and non-evolutionary model.

Lensing increases the apparent brightness of a quasar causing over-representation of multiply imaged quasars in a flux-limited sample. This effect is called the magnification bias. The bias factor for a quasar at redshift z with apparent magnitude m is given by[51–55]

$$\mathbf{B}(m,z) = M_0^2\, B(m,z,M_0,M_2), \qquad (33)$$

where

$$B(m, z, M_1, M_2) = 2\,(\frac{dN_Q}{dm})^{-1} \int_{M_1}^{M_2} \frac{dM}{M^3}\, \frac{dN_Q}{dm}(m + 2.5\log(M), z). \tag{34}$$

We can allow the upper magnification cut off M_2 to be infinite, though in practice we set it to be $M_2 = 10^4$. M_0 is the minimum magnification of a multiply imaged source and for SIS model $M_0 = 2$.

We use Kochanek's "best model"[55] as quasar luminosity function:

$$\frac{dN_Q}{dm}(m, z) \propto (10^{-a(m-\overline{m})} + 10^{-b(m-\overline{m})})^{-1}. \tag{35}$$

where the bright-end slope a and faint-end slope b are constants, and the break magnitude $\overline{m}$ evolves with redshift:

$$\overline{m} = \begin{cases} m_o + (z-1) & \text{for } z < 1, \\ m_o & \text{for } 1 < z \le 3, \\ m_o - 0.7(z-3) & \text{for } z > 3. \end{cases} \tag{36}$$

Fitting this model to the quasar luminosity function data in Hartwich & Schade[56] for $z > 1$, Kochanek finds that "the best model" has $a = 1.07$, $b = 0.27$, and $m_o = 18.92$ at B magnitude.

The magnitude corrected probability, p_i, for the quasar i with apparent magnitude m_i and redshift z_i to get lensed is:

$$p_i = \tau(z_i)\mathbf{B}(m_i, z_i). \tag{37}$$

Since the observations have finite resolution and dynamic range, we must include a selection function to correct the statistics for observational limitations. For the SIS model the selection effects can be characterised by the maximum magnitude difference that can be detected for two images separated by θ, $\Delta m(\theta)$. The selection function corrected probabilities are[55]

$$p_i^{'}(m,z) = p_i \int \frac{d\phi\, p_c(\phi) B(m,z,M_f(\phi),M_2)}{B(m,z,M_0,M_2)}, \tag{38}$$

and

$$p_{ci}^{'} = p_{ci}(\phi)\, \frac{p_i}{p_i^{'}}\, \frac{B(m,z,M_f(\phi),M_2)}{B(m,z,M_0,M_2)}, \tag{39}$$

where

$$p_c(\phi) = \frac{1}{\tau(z_S)} \int_0^{z_S} \frac{d^2\tau}{dz_L d\phi}\, dz_L, \tag{40}$$

$$M_f(\phi) = \; M_0\, (\tfrac{f+1}{f-1}) \;\; \text{for } f > 1, \tag{41}$$

and

$$f = f(\phi) = 10^{0.4\Delta m(\phi)}. \tag{42}$$

Equation (39) defines the configuration probability. It is the probability that the lensed quasar i is lensed with the observed image separation. $p_{ci}(\phi)$ in eq. (39) is $p_c(\phi)$ evaluated for the observed image separation. In our calculations we take $\Delta m = 0.5$.[10]

4.2 *Testing the models against observations*

We perform maximum-likelihood analysis to determine the confidence level of w and Ω_m for the case where comoving number density of lenses is constant and for the two evolutionary models, namely, the VG model and the fast-merging model.

The likelihood function is

$$L = \prod_{i=1}^{N_U} (1 - p_i^{'}) \prod_{k=1}^{N_L} p_k^{'} \, p_{ck}^{i}. \tag{43}$$

Here N_L is the number of multiple-imaged lensed quasars, N_U is the number of unlensed quasars, $p_k^{'}$, the probability of quasar k to get lensed, is given by eq. (38) and p_{ck}^{i}, the configuration probability, is given by eq. (39). We use the same quasar sample as used by Cheng and Krauss.[57] We consider five surveys: Large Bright Quasar Survey (LBQS),[59] HST snapshot survey,[60] Crampton survey,[61] Yee survey[62] and Surdej survey.[63] We considered a total of 1193 ($z > 1$) high luminosity optical quasars which include 5 lenses. The lens surveys and quasar catalogs usually use V magnitudes, so we transform m_V to a B-band magnitude using $B - V = 0.2$ as suggested by Bachall et al. (1992).[58]

5 Results And Discussion

The likelihood function as defined in eq. (43) is a function of the parameters Ω_m and w. We allow parameters Ω_m and w to vary in the range $0 \leq \Omega_m \leq 1$ and $-1 \leq w \leq 0$ and find maximum of the likelihood L_{max}. The logarithm of the ratio of the likelihood to its maximum $-2\ln(L/L_{max})$ is asymptotically distributed like a χ^2 distribution with the degrees of freedom equal to the parameters involved. In the figures we present likelihood function as a function of two parameters and therefore the 68%(1σ) and 95.4% (2σ) confidence levels are defined by the contours on which L are 31.6% and 4.5% of L_{max} respectively. For one parameter fitting, $-2\ln(L/L_{max})$ is distributed like a χ^2 distrbution with one degree of freedom. The 68% and 95.4% confidence limits on the parameter are where L is 60.6% and 14% of L_{max} respectively.

• **No-Evolution Model** Fig. 1 shows countours of constant likelihood (95.4% and 68%) in two parameter space (w,Ω_m). The best fit (L_{max}) occurs for $w = -0.5$ and $\Omega_m = 0.0$. We see that $w \leq -0.24$ and $\Omega_m \leq 0.48$ at 1σ (68% confidence level). For the case of constant Λ i.e. $w = -1.0$, L_{max} occurs for $\Omega_m = 0.24$ ($\Omega_\Lambda = 0.76$) and $0.21 \leq \Omega_m \leq 0.28$ at 1σ. Waga & Miceli[10] have done a similar study with a sample of 867 HLQ and for one parmeter fit they get $\Omega_\Lambda \leq 0.76$ at 2σ, while we get $0.43 \leq \Omega_\Lambda \leq 0.92$ at 2σ. For one parameter fit the constraints on Ω_m are quite strong.

• **VG Model** Fig. 2 summarises results for the VG Model. The maximum of likelihood occurs for $w = -0.47$ and $\Omega_m = 0.0$ with $w \leq -0.2$ and $\Omega_m \leq 0.58$ at 1σ. For constant Λ case, L_{max} occurs for $\Omega_m = 0.27$. Also $0.19 \leq \Omega_m \leq 0.39$ at 1σ.

Model	Fit	Best Fit w, Ω_m	68% CL	95.4% CL
No-Evolution	A	-0.5, 0.0	$w \leq -0.24$ $\Omega_m \leq 0.48$	$w \leq -0.01$ $\Omega_m \leq 0.97$
	B	0.24	$0.21 \leq \Omega_m \leq 0.28$	$0.08 \leq \Omega_m \leq 0.68$
VG	A	-0.47, 0.0	$w \leq -0.2$ $\Omega_m \leq 0.58$	
	B	0.27	$0.18 \leq \Omega_m \leq 0.39$	$0.09 \leq \Omega_m \leq 0.8$
Fast Merging	A	-0.33, 0.0	$w \leq -0.02$ $\Omega_m \leq 0.93$	
	B	0.43	$0.3 \leq \Omega_m \leq 0.6$	$\Omega_m \geq 0.14$

Table 1: Here Fit A refers to the two parameter fit and B refers to one parameter fit i.e. when $w = -1$.

• **Fast Merging Model** Fig. 3 shows contours of constant likelihood (68% and 95.4%) for the Fast Merging Model of galaxy evolution. The best fit occurs for $w = -0.33$ and $\Omega_m = 0.0$. For this model of galaxy evolution the constraints are very weak. We have $w \leq -0.02$ and $\Omega_m \leq 0.93$ at 1σ. For one parameter fit i.e. for the case of constant Λ, L_{max} occurs for $\Omega_m = 0.43$ and $0.30 \leq \Omega_m \leq 0.60$ at 1σ.

Table 1 summarises the results for the three models.

Torres and Waga,[7] Waga and Miceli[10] have done a similar study for decaying Λ cosmologies ($\Lambda \sim a^{-m}$) without taking evolution of galaxies into account. They point out the fact that the constraints put by gravitational lens statistics are very weak in these cosmologies. The main aim of our paper is to see how these constraints change when we take galaxy evolution into account.

In all the three models, we find that constraints are weaker for the case of a two parameter fit as compared to the case of a one parameter fit. This is because as w increases, the distance to an object at redshift z decreases which in turn decreases the probability of lensing. As a result the constraints on the cosmological parameters (w,Ω_m) are weaker. Fig. 4 illustrates how optical depth decreases with increasing w. In the case of two paramter fit, we observe that higher values of w permit only low values of Ω_m.

We observe that the constraints on w and Ω_m obtained in the case of evolutionary models are weaker than those obtained in the case of no-evolution model. Similarly for a one parameter fit ($w = -1$ case), the constraints on Ω_m are weaker in the evolutionary madels. We believe that this can be due the fact that the effect of evolution in these two models is to decrease the probability of lensing.[64] Jain *et al.*[65] constrained Ω_Λ (constant Λ) for flat cosmologies taking into account galaxy evolution. They constrain it using the information of number of lensed quasars and conclude that evolution permits a larger value of Ω_Λ. They also find that in their formalism $\gamma = 4$ masks the difference which evolution of galaxies make. For constant Λ case, our analysis shows that the evolutionary models permit higher values of Ω_m. We would also like to point out that the present formalism allows us to study the difference which evolution of galaxies make, with $\gamma = 4$. We conclude that the results crucially depend on the information about image separation and hence its inclusion is very important.

For the case of no-evolution model, we find that our constraints are stronger than that of Waga and Miceli. We have used a bigger quasar sample. The quasar sample used plays a crucial role in the determination of paramters. We would like to point out that our results differ from the results of Waga & Miceli[10] also because we have performed integration over ϕ in eq. (38). This gives us a factor of $(8\pi(\frac{v^*}{c})^2)^{-1}$ when we perform the integration over θ as is done by Waga & Miceli[10] in equation (3.10) of their paper. Further, for drawing countours of constant likelihood in two dimensions, they use $\frac{L}{L_{max}} = 0.606$ for 68% confidence level and $\frac{L}{L_{max}} = 0.135$ for 95.4%. We define 68% and 95.4% confidence levels in two dimensions as contours for which likelihoods are 31.6% and 4.5% of L_{max} as suggested by Kochanek (1993)[54] and Lampton *et al.* (1976).[66]

In this work we highlight how inclusion of galaxy evolution can change the constraints on the cosmological parameters (w,Ω_m) obtained from lensing statistics. We find that the constrains obtained in the case of VG model and Fast Merging Model are weaker than those obtained in the no-evolution model. Nevertheless we can't ignore galaxy evolution while using lensing statistics as a tool to probe the universe. In this study we consider those galaxy evolutionary models which are based on the assumption that the total comoving mass is conserved during mergers. There is a possibility that the total comoving mass isn't conserved when mergers take place. Work is under progress to see how lensing statistics and cosmological parameters are affected when total comoving mass density of lenses are not conserved.

Acknowledgements

We are thankful to Yu-Chung N. Cheng for providing us the quasar sample. A. Dev thanks University Grant Commmission of India for providing research fellowship.

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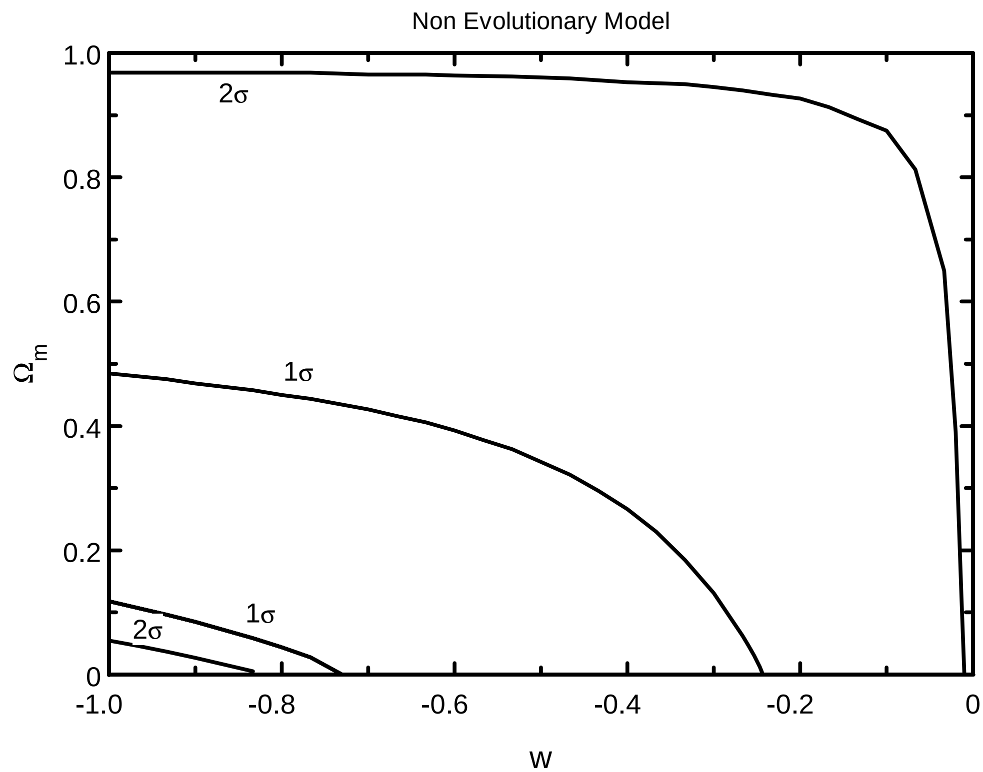

Figure 1: Contours of constant likelihood (68% and 95.4%) for No-Evolution Model for lensing galaxies

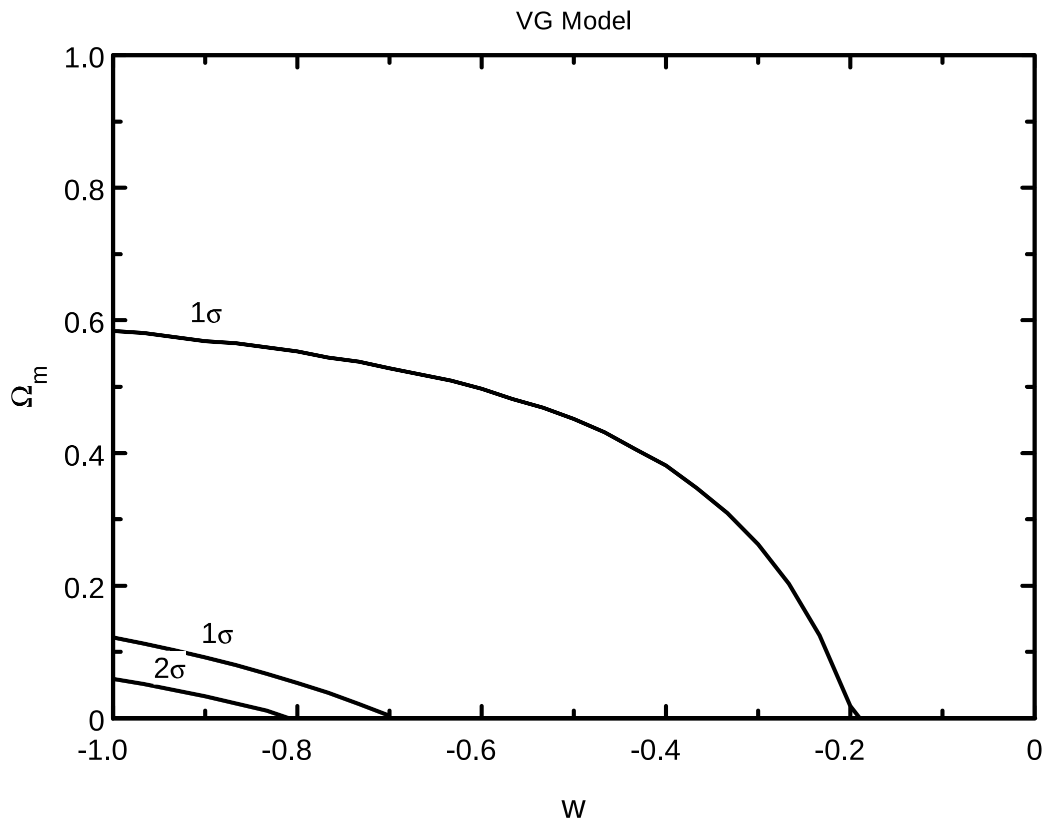

Figure 2: Contours of constant likelihood (68% and 95.4%) for Volmerange & Guiderdoni Model of galaxy evolution.

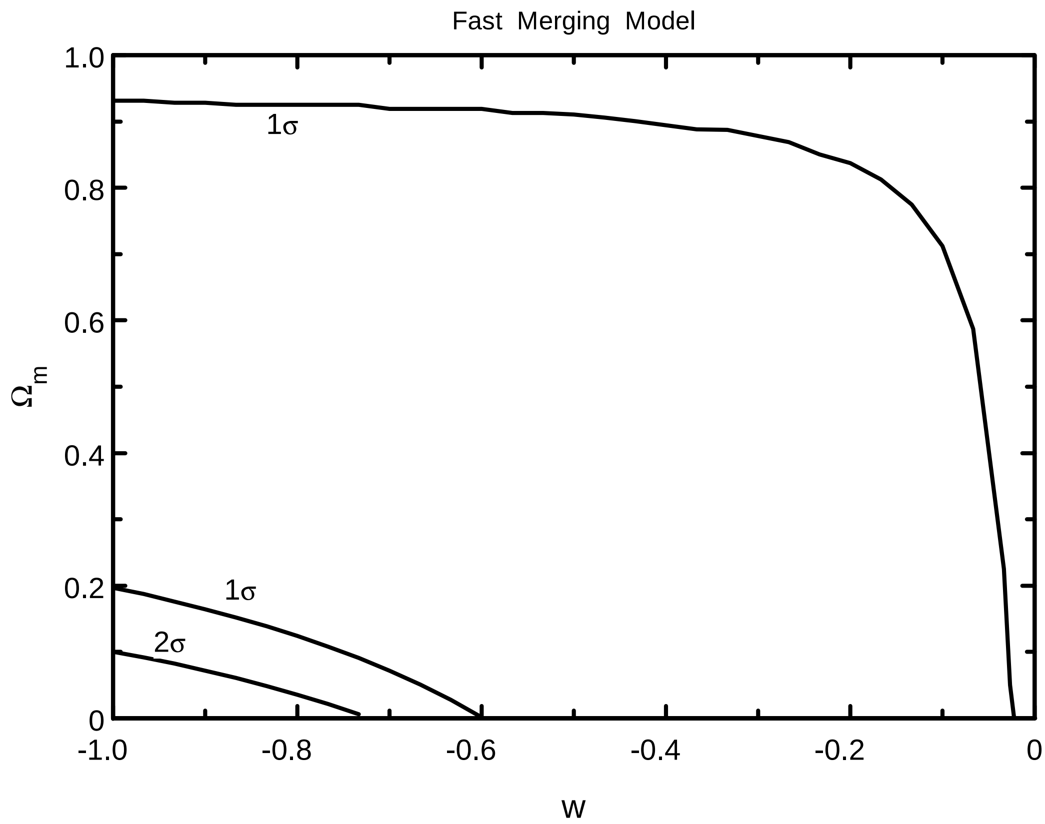

Figure 3: Contours of constant likelihood (68% and 95.4%) for Fast Merging Model of galaxy evolution.

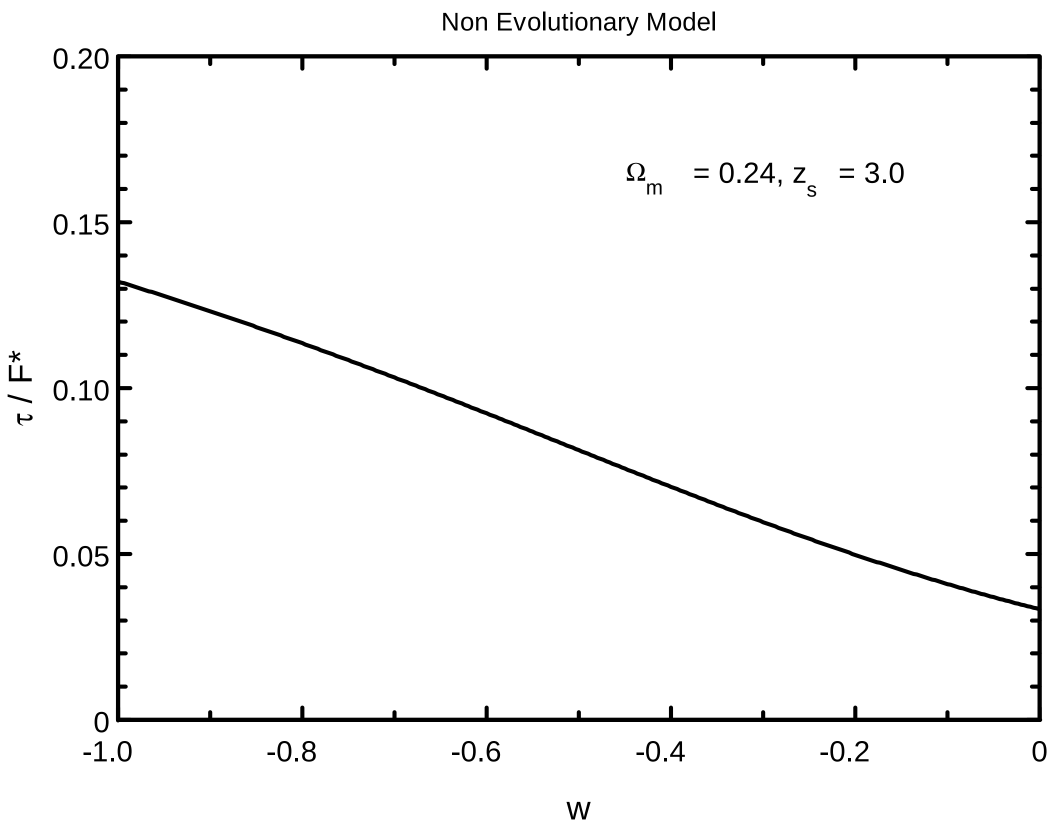

Figure 4: Probability of lensing as a function of w