

CHARACTERISTIC PREDICTIONS OF TOPOLOGICAL SOLITON MODELS

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The characteristic predictions of chiral soliton models - the Skyrme model and its extensions - are discussed. The chiral soliton models predictions of low-lying dibaryon states qualitatively agree with recent evidence for the existence of narrow dibaryons in reactions of inelastic proton scattering on deuterons and double photon radiation $pp \rightarrow pp\gamma\gamma$. The connection between magnetic momentum operators and tensors of inertia valid for arbitrary $SU(2)$ skyrmion configurations allows to estimate the electromagnetic decay width of some states of interest. Another kind of predictions are multibaryons with nontrivial flavour - strangeness, charm or bottom, which can be found, in particular, in high energy heavy ions collisions. It is shown that the large B multiskyrmions given by rational map ansatz can be described within the domain wall approximation, or as spherical bubble with energy and baryon number density concentrated at its boundary.

1 Introduction

The chiral soliton approach provides a very economical method of description of baryonic systems with different baryon numbers, starting with few basic concepts and ingredients incorporated in the model lagrangian [1, 2]. The latter is truncated lagrangian of effective field theories widely used for description and explanation of low-energy meson and baryon interactions [3]. Baryons or baryonic systems appear within this approach as quantized solitonic solutions of equations of motion, characterized by the so called winding number or topological charge. If the concept of topological soliton models is accepted and baryons are skyrmions indeed, then it is clear why there is isospin in the Nature: the number of generators of the $SU(2)$ isospin group, 3, coincides with the number of the space dimensions, thus allowing for the correlation between $SU(2)$ chiral fields and space coordinates resulting in appearance of topological solitons.

As it was found numerically, the chiral field configurations of lowest energy possess different topological properties - the shape of the mass and B -number distribution - for different values of B . It is a sphere for $B = 1$ hedgehog [1], a torus for $B = 2$ [4], tetrahedron for $B = 3$, cube for $B = 4$ [5], and higher polyhedrons for greater baryon numbers [5, 6, 7]. A paradoxical feature of the whole approach is that the baryons/nucleons individuality is absent in the lowest energy static configurations (note, that any of known lowest energy configurations can be made of a number of slightly deformed tori). It is believed that the standard picture of nuclei should appear when non-zero modes motion - vibration, breathing - are taken into account. To get idea about the relative position of states with different quantum numbers - spin, isospin, flavor, $SU(3)$ representation, etc., it is necessary to calculate zero-mode quantum corrections to the energy of baryonic system. Corrections of this kind have been calculated first for the configurations of "hedgehog" type [8], later for configurations with axial symmetry [9, 10] and for more general configurations, in $SU(2)$ [11] as well as $SU(3)$ cases [12, 13].

The chiral soliton approach provides the concept of nuclear matter different from that widely accepted as being constructed from separate nucleons, only. To find the "smoking gun" for this unusual concept it is necessary to find some states which cannot be made of separate nucleons because, e.g. of the Pauli exclusion principle. The simplest possibility is

to consider the $B = 2$ system where the Pauli principle strictly and unambiguously forbids definite sets of quantum numbers for the system consisting of separate nucleons.

Here we discuss first the $SU(2)$ case (next Section) where supernarrow low lying dibaryons were predicted [14] and estimate their electromagnetic decay width. Further the $SU(3)$ extension of the chiral soliton model is considered and estimates for spectra of multi-baryons with flavour (strangeness, charm or bottom quantum number) made previously are extended up to highest baryon numbers where the necessary theoretical information on multiskyrmions is available [7]. A simplified model for large B multiskyrmions given by rational maps [15] is presented which allows us to establish the connection with the domain wall or bag approximation (Section 4). The technical details necessary for calculations are available in literature, and here some of them given in Appendices, where several statements valid for any chiral solitons are proved and useful expressions for $SU(2)$ skyrmion tensors of inertia still lacking in the literature are presented.

2 Narrow dibaryons below $NN\pi$ threshold

The topological chiral solitons (skyrmions) are classical configurations of chiral fields incorporated in unitary matrix $U \in SU(2)$ or $SU(3)$ and characterized by topological, or winding number identified with baryon number B . The classical energy (mass) of these configurations M_{cl} is found usually by minimization of energy functional depending on chiral fields. As any extended object skyrmions possess also other characteristics like moments of inertia Θ (tensors of inertia in general case, see Appendix A), mean square radii of mass and baryon number distribution, etc. The quantization of zero modes of chiral solitons allows to obtain the spectrum of states with different values of quantum numbers: spin, isospin, strangeness, etc. [8]-[13]. This approach allows for quite reasonable description of various properties of baryons, nucleons and hyperons, therefore, it is of interest to consider predictions of the models of this kind for baryonic systems with $B \geq 2$. The energy of $SU(2)$ quantized states with axial symmetry can be presented as [9, 10]

$$E = M_{cl} + \frac{I(I+1)}{2\Theta_I} + \frac{J(J+1)}{2\Theta_J} + \frac{J_3^2}{2B^2\Theta_3} \left(1 - \frac{\Theta_3}{\Theta_I} - B^2 \frac{\Theta_3}{\Theta_J} \right) \quad (1)$$

Here I and J are isospin and spin of the system, J_3 is body fixed 3-d component of the angular momentum which can be considered as an additional internal quantum number of the system. $B = n$ - azimuthal winding number for the lowest energy axially symmetrical configurations. This formula, being obtained rigorously from the model lagrangian [9, 10], is very transparent in its physical meaning. The technical details beginning with known lagrangian of the Skyrme model, expressions for M_{cl} , tensors of inertia and some other formulas can be found in Appendix A.

The (generalized) axial symmetry of the configuration with $B = 2$ leads to definite constraint on the body-fixed 3-d components of the isospin and angular momentum: $J_3 = -nI_3 = -nL$ [9, 10]. As a consequence of this, the states with $I = 1$ and $J = 0$, or $I = 0$, $J = 1$, and also $I = J = 1$ should have $I_3^{bf} = J_3^{bf} = L = 0$. Therefore, the last term in (1) proportional to $J_3^{bf\ 2}$ is absent in these cases. Since the parity of configuration equals to $P = (-1)^L$ [10], all states mentioned above have positive parity. The state with $I = 0$, $J = 2$ can have $I_3^{bf} = J_3^{bf} = 0$ as well as, e.g., $I_3^{bf} = L = 1$, $J_3^{bf} = -2$. At large B by special reasons (see Appendix A) also only the first two terms in (1), $\sim I(I+1)$ and $\sim J(J+1)$ are important in quantum correction to the energy.

As it was noted long ago [9] the quantum correction for the deuteron- like state with $I = 0, J = 1$, $E_d^{rot} = 1/\Theta_J(B = 2)$ is by about $\sim 30\text{Mev}$ smaller than that of the "quasi-deuteron" state with $I = 1, J = 0$, $E_d^{rot} = 1/\Theta_I(B = 2)$. This takes place for all known variants of the model, without any tuning of parameters, therefore, it can be considered as intrinsic property of chiral soliton models originating from effective field theories. Further investigations of nonzero modes of two-nucleon system, not all but many of them, have shown that the binding energy of the deuteron can be reduced to $\sim 6\text{Mev}$ [16] if it is considered as a difference between states with deuteron and quasideuteron quantum numbers. Previously and here we consider the differences of energies of quantized states because they are free of many uncertainties due, e.g., to unknown loop corrections to the masses of skyrmions (see [17, 18] and discussions below).

According to expression (1) dibaryons are predicted decoupled from 2- nucleon channel as a consequence of the Pauli principle [14]. For example, the state with isospin $I = J = 1$, positive parity and the energy below the threshold for the decay into $NN\pi$ with $E_D^{rot} = 1/\Theta_J(B = 2) + 1/\Theta_I(B = 2)$. This dibaryon cannot be seen in nucleon-nucleon interactions directly, but can be observed in reaction $NN \rightarrow NN\gamma\gamma$, where one photon is necessary to produce D and the second one appears from the decay of D : e.g., $pp \rightarrow D^{++}\gamma \rightarrow pp\gamma\gamma$. The chiral soliton models predict the state D with isospin $I = J = 1$ at the energy about $50-60\text{Mev}$ above the NN threshold [14].

In the paper [10] it was shown that the states with even sum $I + J$ (0, 2, etc.) and positive parity are forbidden by the Finkelstein- Rubinstein type constraints which appear as a consequence of requirement that the configuration can be presented as a system of two unit hedgehogs at large relative distances, and these unit skyrmions possess fermionic properties. It means, that the configurations which cannot be considered as consisting of two nucleons, were ignored in [10]. Opposite to this, we abandoned this requirement [14]. It should be noted also that the state with $I = 0, J = 2$ which was forbidden in [10] can be in fact the 3D_2 state of two nucleons and should not be forbidden by FR - constraint. Therefore, this particular case should be analyzed more carefully.

It is possible to estimate the width of the radiative decay $D \rightarrow NN\gamma$. Electromagnetic nucleon formfactors can be described quite well within Skyrme soliton model in wide interval of momentum transfers [19], reasonable agreement with data takes place for deuteron and $2N$ - system [10], therefore, one can expect reasonable predictions for systems with greater baryon numbers or with unusual properties. The dimensional estimate of narrow dibaryon decay width was made in [14] providing the lower bound for the decay width of few eV . To make more realistic estimate one can consider magnetic type transition $D \rightarrow NN\gamma$ or $d\gamma$. The amplitude of the direct process due to magnetic dipole transition can be written as

$$M_{D \rightarrow NN\gamma} = ie \tilde{\mu}_{D \rightarrow NN} \epsilon_{ikl} F_{ik} \Psi_l^D \phi_1^\dagger \phi_2 \quad (2)$$

where $\tilde{\mu}$ is the value of the transition magnetic moment, we assume that μ is of the order of μ_p , $F_{ik} = e_i q_k - e_k q_i$ - the electromagnetic field strength, Ψ_l^D , ϕ_1 and ϕ_2 are the wave functions of the dibaryon and nucleons. For the width of such direct decay we obtain then

$$\Gamma_{D \rightarrow NN\gamma} = \alpha \Delta M^2 \frac{\tilde{\mu}_{D \rightarrow NN}^2}{945\pi^2} (\Delta/M)^{7/2} \quad (3)$$

which is numerically less than 0.1eV for $\mu \sim \mu_p - \mu_n \simeq 4.7/(2M_N)$, $\Delta = M_D - 2M$ is the energy release, or the maximal energy of emitted photon. This estimate agrees with that made previously [14], but the final state interaction could increase it by several orders of magnitude.

To take it into account roughly one should consider the transition $D \rightarrow d'$ where d' is spin zero quasideuteron, or $D^+ \rightarrow d$. At this point the important statement is that the isovector magnetic transition operator for any skyrmion is connected simply with its mixed, or interference tensor of inertia Θ_{ab}^{int} . This statement, known for some particular cases [8, 10] is proved in Appendix B for arbitrary skyrmions and for any type of chiral soliton models:

$$\tilde{\mu}_i^a = -\frac{1}{2}R^{aj}(A)\Theta_{jk}^{int}O_i^k(A'), \quad (4)$$

where $R^{aj} = D_{aj}^1 = Tr(A^\dagger \tau^a A \tau^j)/2$ and O_i^k are the final rotation matrices, a is isotopical (octet in $SU(3)$) index, and for electromagnetic interaction we should take $a = 3$. Θ_{jk}^{int} is presented in Appendix A.

For configurations with generalized axial symmetry, as well as for several known multiskyrmions, only diagonal elements of Θ^{int} are different from zero, moreover, only the (33) component remains in the case of axial symmetry, and we have

$$\tilde{\mu}_i^3 = -\frac{1}{2}R^{33}(A)\Theta_{33}^{int}O_i^3(A'), \quad (5)$$

$\Theta_{33}^{int} = 2\Theta_{33}^I = 14.8 Gev^{-1}$ for $B = 2$ and accepted values of model parameters, see also **Table 1** below. To get numerical values of the transition magnetic moments one should calculate the matrix elements of rotation matrices between the wave functions of initial and final states which are equal in terms of final rotation matrices $D_{I_3,L}^I$, see e.g. [20]

$$\Psi_{I,I_3;J,J_3}^D = \sqrt{\frac{2I+1}{8\pi^2}}D_{I_3L}^I \sqrt{\frac{2J+1}{8\pi^2}}D_{J_3,-2L}^J, \quad (6)$$

and we have for D state $I = J = 1$, $L = 0$, for the final d' state also $I = 1$, and $J = 0$. Since $R^{33} = D_{00}^1$ the isotopical part of the matrix element for $D \rightarrow d'$ transition is proportional to

$$\langle D_{I_3,0}^1 D_{00}^1 D_{I_3,0}^1 \rangle = \int D_{I_3,0}^1 D_{00}^1 D_{I_3,0}^1 d\nu = C_{1,0;1,I_3}^{1,I_3} C_{1,0;1,0}^{1,0}/3. \quad (7)$$

The Clebsch-Gordan coefficient $C_{1,0;1,0}^{1,0} = 0$, therefore $D \rightarrow d'$ transition magnetic moment equals to zero for all states including D^{++} and D^0 , not only for $D^+ \rightarrow d'^+$ which is trivial, and this is a consequence of symmetry property of rotator wave function with $L = 0$.

For the transition $D^+ \rightarrow d\gamma$ the isotopical part of the matrix element differs from zero, $\langle D_{0,0}^1 D_{00}^1 D_{00}^0 \rangle = 1/3$, but the angular momentum part proportional to $\langle D_{J_3,0}^1 D_{00}^1 D_{J_3,0}^1 \rangle$ equals to zero again. However, the decay $D^+ \rightarrow np$ is possible as a result of isospin violation in the second order in electromagnetic interaction, due to virtual emission and reabsorption of the photon and due to isospin violation by the mass difference of u and d quarks. The order of magnitude estimate of the width of this decay due to the virtual electromagnetic process is

$$\Gamma_{D \rightarrow pn} \simeq \alpha^2 \frac{M}{4\pi} \sqrt{\Delta/M} \quad (8)$$

which is about $\sim 1KeV$. It should be noted here that for the components of D with charge $+2$ or 0 the decay into pp or nn final states is forbidden strictly, due to the rigorous conservation of angular momentum and the Pauli principle.

For transition $D^{++} \rightarrow pp\gamma$, $D^0 \rightarrow nn\gamma$ as well as $D^+ \rightarrow (pn)_{I=1}\gamma$ the isoscalar magnetic momentum operator gives nonzero contribution. The corresponding matrix element

$$M_{D \rightarrow d'\gamma} = ie \tilde{\mu}_{D \rightarrow d'}^0 \epsilon_{ikl} F_{ik} \Psi_l^D \Psi^{d'\dagger} \quad (9)$$

The approximate relation takes place for rational map parameterization:

$$\tilde{\mu}_3^0 \simeq J_3 \frac{B \langle r_0^2 \rangle}{3\Theta^J}, \quad (10)$$

$\langle r_0^2 \rangle$ is mean square radius of B -number distribution. (10) coincides with result of [8] for $B = 1$ and is close to result of [10] for $B = 2$. The derivation of (10) valid for rational maps parametrization of skyrmions, will be given elsewhere. The coefficient before J_3 in (10) depends remarkably weak on baryon number, as can be established from **Table 1**. However, numerically (10) gives about twice smaller result for $B = 1$ for parameters we take here, than in [8]. As a result we have:

$$\tilde{\mu}_{D \rightarrow d'}^0 \simeq \frac{2 \langle r_0^2 \rangle}{3\Theta^J}, \quad (11)$$

For the decay width we obtain then

$$\Gamma_{D \rightarrow d' \gamma} = \alpha \frac{4\tilde{\mu}_{D \rightarrow d'}^2 \Delta^3}{3} \quad (12)$$

Numerically, $\tilde{\mu}_{D \rightarrow d'} \simeq 0.35 \text{ Gev}^{-1}$, and from (12) $\Gamma_{D \rightarrow d' \gamma} \sim 0.3 \text{ KeV} (\Delta/60 \text{ MeV})^3$. The same estimate is valid for the decay rate of $D^+ \rightarrow np\gamma$ with np -system in isospin $I = 1$ state.

The experimental evidence for the existence of narrow dibaryon D in reaction $pp \rightarrow pp\gamma\gamma$ has been obtained in Dubna [21], although these data have not been confirmed in the Uppsala bremsstrahlung experiment [22]. Even more clear indications for the existence of low lying dibaryons have been obtained in experiment at Moscow meson factory in the reaction $pd \rightarrow pX$ [23]. The checking and confirmation of these results in its importance is comparable with a discovery of new elementary particle. The absence of such states would provide definite restrictions on applicability of the chiral soliton approach and effective field theories.

It should be noted that within the model there is a problem of the lowest state with $I = J = 0$ which should be lower than the deuteron-like state. Therefore, deuteron should decay into this (0,0) state and a photon, but two-nucleon system in singlet 1S_0 state could not decay because $0 \rightarrow 0$ transition is forbidden for electromagnetic interaction. The loop corrections to the energy of states, or Casimir energy [16], are different for states which can go over into two nucleons, and for states which cannot. Their contribution can change the relative position of these states and shift the (0,0) state above the deuteron, but very nontrivial calculation should be made to check this.

Some low-lying states with strangeness are also predicted, which cannot decay strongly due to the parity and isospin conservation in strong interactions [14]. For example, the dibaryon with strangeness $S = -2$, $I = 0$, $J = 1$ and positive parity has energy by $\sim 0.17 \text{ GeV}$ above $\Lambda\Lambda$ threshold [24], and it cannot decay into two Λ -hyperons because of the Pauli principle, and into $\Lambda\Lambda\pi$ final state by isospin conservation. Therefore, the width of electromagnetic decay of such state should be not more than few tenths of KeV . It is, of course, a special case. Other possible states with flavour s , c or b will be discussed in the next section.

The masses of neutron rich light nuclides, such as tetra-neutron, sexta-neutron, etc. can be estimated using formula (1). For multineutron state with $I = B/2$ the rotation energy $E^{rot} = B(B+2)/(8\Theta_I)$, and such nuclides are predicted well above the threshold for the strong decay into final nucleons. With increasing baryon numbers the energies of neutron rich states with fixed difference $N - Z$ become lower, so, their width can be quite small. The mass difference of states with isospin I and ground states with $I = 0$ (for even B) equals to $\Delta E(B, I) = I(I+1)/(2\Theta_{I, B})$. For such pairs of nuclei as $^8\text{Li} - ^8\text{Be}$, $^{12}\text{B} - ^{12}\text{C}$ and $^{16}\text{N} - ^{16}\text{O}$ it equals to $\Delta E(B, 1) = 1/\Theta_{I, B}$ and decreases with increasing B , i.e. atomic number, both theoretically (see **Table 1**. below) and according to data. For $B = 16$ this difference equals to 10.9 MeV in comparison with theoretical value of 15.8 MeV which is not bad for such a crude model.

3 Flavoured multibaryons

Another characteristic prediction is that of multibaryons with different values of flavours, such as strangeness, charm or bottom quantum numbers. The bound state approach of multiskyrmions with different flavours is the adequate method to calculate the binding energies of states with quantum numbers s , c or b . The so called rigid oscillator model is the most transparent and controllable version of this method [25]. The references to pioneer papers can be found also in [26]. The binding energies of flavoured states are predicted smaller than binding energies of ordinary nuclei - for strangeness quantum numbers, and greater - for charm or bottom quantum numbers. Here I present the main results on flavoured multibaryons following the papers [26] and extended to higher values of baryon numbers.

To quantize the solitons in $SU(3)$ configuration space, in the spirit of the bound state approach to the description of strangeness, we considered the collective coordinates motion of the meson fields incorporated into the matrix $U \in SU(3)$, see Appendix A:

$$U(r, t) = R(t)U_0(O(t)\vec{r})R^\dagger(t), \quad R(t) = A(t)S(t), \quad (13)$$

where U_0 is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the left upper corner), $A(t) \in SU(2)$ describes $SU(2)$ rotations, $S(t) \in SU(3)$ describes rotations in the “strange”, “charm” or “bottom” directions, and $O(t)$ describes rigid rotations in real space.

$$S(t) = \exp(i\mathcal{D}(t)), \quad \mathcal{D}(t) = \sum_{a=4, \dots, 7} D_a(t)\lambda_a, \quad (14)$$

λ_a are Gell-Mann matrices of the (u, d, s) , (u, d, c) or (u, d, b) $SU(3)$ groups. The (u, d, c) and (u, d, b) $SU(3)$ groups are quite analogous to the (u, d, s) one. For the (u, d, c) group a simple redefinition of hypercharge should be made. For the (u, d, s) group, $D_4 = (K^+ + K^-)/\sqrt{2}$, $D_5 = i(K^+ - K^-)/\sqrt{2}$, etc., for the (u, d, c) group $D_4 = (D^0 + \bar{D}^0)/\sqrt{2}$, etc.

The angular velocities of the isospin rotations are defined in the standard way: $A^\dagger \dot{A} = -i\vec{\omega}\vec{\tau}/2$. We shall not consider here the usual space rotations explicitly because the corresponding moments of inertia for baryonic systems (BS) are much greater than isospin moments of inertia, see **Table 1.**, and for lowest possible values of angular momentum J the corresponding quantum correction is either exactly zero (for even B), or small.

The field D is small in magnitude, at least, of order $1/\sqrt{N_c}$, where N_c is the number of colours in QCD . Therefore, an expansion of the matrix S in D can be made safely. To the lowest order in field D the Lagrangian of the model (A1) can be written as

$$L = -M_{cl, B} + 4\Theta_{F, B}\dot{D}^\dagger\dot{D} - \left[\Gamma_B\bar{m}_D^2 + \tilde{\Gamma}_B(F_D^2 - F_\pi^2) \right] D^\dagger D - i\frac{N_c B}{2}(D^\dagger\dot{D} - \dot{D}^\dagger D), \quad (15)$$

$\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2$. Here and below D is the doublet K^+ , K^0 (D^0 , D^- , or B^+ , B^0). Θ_F is the moment of inertia for the rotation into the “flavour” direction ($F = s, c$, or b , the index c denotes the charm quantum number, except in N_c):

$$\Theta_{F, B} = \frac{1}{8} \int (1 - c_f) \left[F_D^2 + \frac{1}{e^2} \left((\vec{\partial}f)^2 + s_f^2 (\vec{\partial}n_i)^2 \right) \right] d^3r, \quad (16)$$

where f is the profile function of skyrmion, F_D is flavour decay constant, i.e. decay constant of kaon, or D - meson, or B -meson,

$$\Gamma_B = \frac{F_\pi^2}{2} \int (1 - c_f) d^3r \quad (17)$$

The mass term contribution to static soliton energy is connected with Γ due to relation $M.t. = m_\pi^2 \Gamma/2$. The quantity $\tilde{\Gamma}_B$ enters when flavour symmetry breaking in flavour decay constants is taken into account:

$$\tilde{\Gamma}_B = \frac{1}{4} \int c_f [(\vec{\partial}f)^2 + s_f^2 (\vec{\partial}n_i)^2] d^3r. \quad (18)$$

It is connected with other calculated quantities via relation:

$$\tilde{\Gamma} = 2(M_{cl}^{(2)}/F_\pi^2 - e^2 \Theta_F^{Sk}),$$

where $M_{cl}^{(2)}$ is second order contribution into static mass of the soliton, Θ_F^{Sk} is Skyrme term contribution into flavour moment of inertia. The contribution proportional to $\tilde{\Gamma}_B$ in (15) is suppressed in comparison with the term $\sim \Gamma$ by the small factor $\sim F_D^2/m_D^2$, and is more important for strangeness. The term proportional to $N_c B$ in (15) arises from the Wess-Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavour and antiflavour in general case) [25, 26].

Following the canonical quantization procedure the Hamiltonian of the system, including the terms of the order of N_c^0 , takes the form [25]:

$$H_B = M_{cl,B} + \frac{1}{4\Theta_{F,B}} \Pi^\dagger \Pi + \left(\Gamma_B \bar{m}_D^2 + \tilde{\Gamma}_B (F_D^2 - F_\pi^2) + \frac{N_c^2 B^2}{16\Theta_{F,B}} \right) D^\dagger D + i \frac{N_c B}{8\Theta_{F,B}} (D^\dagger \Pi - \Pi^\dagger D). \quad (19)$$

Π is the momentum canonically conjugate to variable D which describes the oscillator-type motion of the (u, d) $SU(2)$ soliton in $SU(3)$ configuration space. After the diagonalization which can be done explicitly [25], the normal-ordered Hamiltonian can be written as

$$H_B = M_{cl,B} + \omega_{F,B} a^\dagger a + \bar{\omega}_{F,B} b^\dagger b + O(1/N_c), \quad (20)$$

with a^\dagger, b^\dagger being the operators of creation of strangeness, i.e., antikaons, and antistrangeness (flavour and antiflavour) quantum number, $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ being the frequencies of flavour (antiflavour) excitations. D and Π are connected with a and b in the following way [25]:

$$D^i = (b^i + a^{\dagger i})/\sqrt{N_c B \kappa_{F,B}}, \quad \Pi^i = \sqrt{N_c B \kappa_{F,B}} (b^i - a^{\dagger i})/(2i) \quad (21)$$

with $\kappa_{F,B} = [1 + 16(\bar{m}_D^2 \Gamma_B + (F_D^2 - F_\pi^2) \tilde{\Gamma}_B \Theta_{F,B})/(N_c B)^2]^{1/2}$. For the lowest states the values of D are small: $D \sim [16\Gamma_B \Theta_{F,B} \bar{m}_D^2 + N_c^2 B^2]^{-1/4}$, and increase, with increasing flavour number $|F|$ like $(2|F| + 1)^{1/2}$. As was noted in [25], deviations of the field D from the vacuum decrease with increasing mass m_D , as well as with increasing number of colours N_c , and the method works for any m_D (and also for charm and bottom quantum numbers).

$$\omega_{F,B} = N_c B (\kappa_{F,B} - 1)/(8\Theta_{F,B}), \quad \bar{\omega}_{F,B} = N_c B (\kappa_{F,B} + 1)/(8\Theta_{F,B}). \quad (22)$$

As was observed in [26], the difference $\bar{\omega}_{F,B} - \omega_{F,B} = N_c B/(4\Theta_{F,B})$ coincides, to the leading order in N_c with the expression obtained in the collective coordinates approach [24].

The flavor symmetry breaking (FSB) in the flavour decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.22$ and $F_D/F_\pi = 1.7 \pm 0.2$ (we take $F_D/F_\pi = 1.5$ and $F_B/F_\pi = 2$) leads to the increase of the flavour excitation frequencies, in better agreement with data for charm and bottom. It also leads to some increase of the binding energies of BS [26].

B	M_{cl}	$\Theta_F^{(0)}$	Θ_I	$\Theta_{I,3}$	$\bar{\Theta}_J$	Γ	$\tilde{\Gamma}$	$\langle r_0 \rangle$	ω_s	ω_c	ω_b
1	1.702	2.05	5.55	5.55	5.55	4.80	15	2.51	0.309	1.542	4.82
2	3.26	4.18	11.5	7.38	23	9.35	22	3.46	0.293	1.511	4.76
3	4.80	6.34	14.4	14.4	49	14.0	27	4.10	0.289	1.504	4.75
4	6.20	8.27	16.8	20.3	78	18.0	31	4.53	0.283	1.493	4.74
5	7.78	10.8	23.5	19.5	126	23.8	35	5.10	0.287	1.505	4.75
6	9.24	13.1	25.4	27.7	178	29.0	38	5.48	0.287	1.504	4.75
7	10.6	14.7	28.9	28.9	220	32.3	43	5.72	0.282	1.497	4.75
8	12.2	17.4	33.4	31.4	298	38.9	46	6.15	0.288	1.510	4.79
9	13.9	20.5	37.7	37.7	375	46	47	6.49	0.291	1.517	4.77
12	18.4	28.0	48.5	48.5	636	64	54	7.31	0.294	1.526	4.79
13	19.9	30.5	52.0	52.0	737	70	57	7.5?	0.288	1.497	4.70
14	21.5	33.6	56.1	56.1	865	78	59	7.85	0.299	1.536	4.80
16	24.5	38.9	63.1	63.1	1107	91	63	8.31	0.301	1.543	4.81
17	25.9	41.2	66.1	66.1	1219	96	65	8.48	0.300	1.542	4.81
22	33.7	56.0	84.2	84.2	2027	135	73	9.55	0.308	1.560	4.84
32*	49.1	86.7	118	118	4154	218	87	11.3	0.319	1.585	4.84

Table 1. Characteristics of the bound states of skyrmions with baryon numbers up to $B = 22$. The classical mass of solitons M_{cl} is in GeV , moments of inertia Θ_F , Θ_I , $\Theta_{I,3}$, and Θ_J as well as $\langle r_0 \rangle$, Γ and $\tilde{\Gamma}$ - in GeV^{-1} , the excitation frequencies for flavour F , ω_F in GeV . $\langle r_0 \rangle = \sqrt{r_B^2}$ with Θ_J defines the value of isoscalar magnetic momentum of multiskyrmion. For larger baryon numbers, beginning with $B = 9$, calculations are made using rational maps (RM) ansatz. For $B = 32$ it was assumed that the ratio $\mathcal{I}/B^2 = 1.28$ as for RM $B = 22$ skyrmion. The external parameters of the model are $F_\pi = 186 MeV$, $e = 4.12$. The accuracy of calculations is better than 1% for the masses and few % for other quantities.

$\bar{\Theta}_J$ shown in **Table 1** is 1/3 of the trace of corresponding tensor of inertia, see Appendix A. As it can be seen from **Table 1** the flavour excitation energies increase again for $B = 22$, and the important property of binding becomes weaker for largest B . It can be, however, the artefact of the RM approximation discussed in the next Section. In particular, for rational maps solitons with $B \geq 9$ we take as moment of inertia Θ_I and $\Theta_{I,3}$ 1/3 of the trace of corresponding tensor of inertia, see Appendix A.

For large value $F_D/F_\pi = \rho_D$ and mass m_D , the following approximate formula for the flavor excitation frequencies can be obtained:

$$\omega_{F,B} \simeq \tilde{m}_D \left(1 - 2 \frac{\Theta_{F,B}^{Sk}}{\rho_D^2 \Gamma_B} \right) - \frac{N_c B}{2\rho_D^2 \Gamma_B} \quad (23)$$

with $\tilde{m}_D^2 = m_D^2 + F_\pi^2 \tilde{\Gamma}_B / \Gamma_B$. It is clear from (23) that, first, ω 's are smaller than masses of mesons m_D , i.e. the binding takes place always, and it is to large degree due to the contribution of the Skyrme term into the flavour inertia $\Theta_{F,B}^{Sk}$. When $\rho_D \rightarrow \infty$, $\omega_F \rightarrow m_D$. Since the ratios $\tilde{\Gamma}_B / \Gamma_B$ decreases with increasing B and $\Theta_{F,B} / \Gamma_B$ increases when B increases from 1 to 4 - 7, the energies $\omega_{F,B}$ decrease for these B -numbers, therefore, it leads to increase of binding of flavoured mesons by $SU(2)$ solitons with increasing B up to $B \sim 4 - 7$. However, for $B = 22$ and 32 the ratio $\Theta_{F,B} / \Gamma_B$ is smaller than for $B = 1$, and, indeed, the ω 's are the same and even larger than for $B = 1$.

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$
2	-0.047	-0.03	0.02	-0.053	-0.07	0.02
3	-0.042	-0.01	0.04	-0.036	-0.03	0.06
4	-0.020	0.019	0.06	-0.051	0.022	0.10
5	-0.027	0.006	0.05	-0.063	0.001	0.08
6	-0.019	0.016	0.05	-0.045	0.023	0.10
7	-0.016	0.021	0.06	-0.041	0.033	0.11
8	-0.017	0.014	0.02	-0.040	0.021	0.03
9	-0.023	0.005	0.03	-0.10	-0.003	0.06
12	-0.021	0.003	0.02	-0.09	-0.004	0.04
17	-0.027	-0.013	0.00	-0.11	-0.03	-0.00
22	-0.034	-0.028	-0.03	-0.14	-0.06	-0.03

Table 2. The binding energy differences $\Delta\epsilon_{s,c,b}$ are the changes of binding energies of lowest BS with flavour s, c or b and isospin $I = T_r + |F|/2$ in comparison with the usual u, d nuclei, for the flavour numbers $S = -1, -2, c = 1, 2, b = -1$ and -2 (see Eq. (24)). The $SU(3)$ multiplets are $(p, q) = (0, 3B/2)$ for even B and $(p, q) = (1, (3B - 1)/2)$ for odd B .

The binding energies differences between flavoured multibaryons and ordinary nuclei in the rigid oscillator approximation are given by the formula:

$$\Delta\epsilon_{B,F} = |F| \left[\omega_{F,1} - \omega_{F,B} - \frac{3(\kappa_{F,1} - 1)}{8\kappa_{F,1}^2 \Theta_{F,1}} - \frac{T_r(\kappa_{F,B} - 1)}{4\kappa_{F,B} \Theta_{F,B}} - \frac{(|F| + 2)(\kappa_{F,B} - 1)^2}{8\kappa_{F,B}^2 \Theta_{F,B}} \right], \quad (24)$$

and the lowest $SU(3)$ multiplets are considered with isospin of flavourless component $T_r = 0$ for even B and $T_r = 1/2$ for odd B . This formula is correct for $|F| = 1$ and for any $|F|$ if the baryon number is large enough to ensure the isospin balance.

The values of $\Delta\epsilon$ shown in **Table 2.** should be considered as an estimate. They illustrate the restricted possibilities of RM approximation for large B multiskyrmions.

Isosinglet BS , in particular those with $|F| = B$ are of special interest. As it was argued in [26] such states do not belong to the lowest possible $SU(3)$ irreps, they should have $T_r = |F|/2$. It makes sense to calculate the difference of binding energy of such state and the minimal state (p^{min}, q^{min}) with zero flavour which we identify with usual nucleus (ground state):

$$\Delta\epsilon_{B,F} = |F| \left[\omega_{F,1} - \omega_{F,B} - \frac{3(\kappa_{F,1} - 1)}{8\kappa_{F,1}^2 \Theta_{F,1}} + \frac{(|F| + 2)(\kappa_{F,B} - 1)}{8\kappa_{F,B}^2 \Theta_{F,B}} \right] - \frac{1}{2\Theta_{T,B}} [|F|(|F| + 2)/4 - T_r^{min}(T_r^{min} + 1)] \quad (25)$$

where $T_r^{min} = 0$, or $1/2$ as before.

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$	$\Delta\epsilon_{s=-3}$	$\Delta\epsilon_{c=3}$	$\Delta\epsilon_{b=-3}$	$\Delta\epsilon_{s=-B}$
2	-	-	-	-0.075	-0.03	0.02	-	-	-	-0.07
3	0.000	0.034	0.07	-	-	-	-0.08	0.002	0.09	-0.08
4	-	-	-	-0.047	0.030	0.09	-	-	-	-0.13
5	-0.003	0.032	0.06	-	-	-	-0.06	0.035	0.12	-0.15
6	-	-	-	-0.044	0.025	0.09	-	-	-	-0.21
7	0.000	0.040	0.07	-	-	-	-0.04	0.068	0.15	-0.20
8	-	-	-	-0.039	0.023	0.03	-	-	-	-0.28
12	-	-	-	-0.046	0.00	0.03	-	-	-	-0.50
17	-0.020	-0.01	-0.00	-	-	-	-0.081	-0.04	-0.01	-0.82
22	-	-	-	-0.073	-0.06	-0.06	-	-	-	-1.3
32*	-	-	-	-0.088	-0.11	-0.13	-	-	-	--

Table 3. The binding energies differences of lowest flavoured BS with isospin $I = 0$ and the ground state with the same value of B and $I = 0$ or $I = 1/2$. The first 3 columns are for $|F| = 1$, the next 3 columns - for $|F| = 2$, and the next 3 - for $|F| = 3$. The state with the value of flavour $|F|$ belongs to the $SU(3)$ multiplet with $T_r = |F|/2$. In the last column the binding energies differences are shown for the isoscalar electrically neutral states with $S = -B$. For $|F| \geq 3$ all estimates are very approximate.

According to **Table 3** the total binding energy of state e.g. with $B = 22$ and $S = -2$ is by 73 Mev smaller than that of nucleus $A = 22$, so it should be well bound. The model used here is too crude for large values of flavour, and results obtained can be used only for illustration and as a starting point for further investigations. Results similar to those described in this section are obtained also in other versions of the model [27], in particular in the quark-meson soliton model [28]. For baryon numbers $B = 3, 4$ estimates of spectra of baryonic systems with charm quantum number were made in [29] within conventional quark model. They are in fair agreement with ours.

In the channel with $B = 2$ the near-threshold state with strangeness $S = -1$ was observed long ago in the reaction $pp \rightarrow p\Lambda K^+$ [30] and confirmed recently in COSY experiment [31]. Similar near-threshold $\Lambda\Lambda$ state was observed by KEK PS E224 collaboration [32]. The Skyrme model explains these near-threshold states with $B = 2$, and predicts similar states for greater values of the B -number. For some values of B , beginning with $B \geq 5, 6$ such states with several units of strangeness can be stable relative to strong interactions. Due to well known relation between charge, isospin and hypercharge of hadrons, $Q = I_3 + (B + S)/2$, the BS with several units of strangeness can appear as negatively charged nuclear fragments. For even B and minimal multiplets $(p, q) = (0, 3B/2)$, strangeness $S = -2I$, and condition when $Q = -1$ fragment appears first is $-1 = S + B/2$, or $-S = B/2 + 1$. For $B = 6$ it is $S = -4$, for $B = 8$, $S = -5$, etc. For odd B the $Q = -1$ state should have strangeness $-S = (B - 1)/2 + 1$, i.e. $-3, -4$ and -5 for $B = 5, 7$ and 9 , etc.

The negatively charged long living nuclear fragment with mass about 7.4 Gev observed in NA52 CERN experiment in $Pb + Pb$ collision at the energy of 158 A Gev [33] can be, within the chiral soliton models, a fragment with $B = 7$ or 6 and strangeness $S = -4$ or $-5, -6$. Confirmation of this result as well as searches for other negatively charged fragments would be of great importance. For charm or bottom quantum numbers the binding energies are greater, but to observe such states one needs considerably higher incident energies.

4 Large B multiskyrmions from rational maps in the domain wall approximation

The rational map ansatz for skyrmions proposed in [15] and widely used now, also in present paper, simplified considerably the treatment of multiskyrmions, and, at the same time, it leads to the picture of multibaryon system at large B which is, probably, incompatible with a picture for ordinary nuclei. To clarify this point, we consider here large B multiskyrmions in some kind of a toy model - in a domain wall approximation, which gives, however, quite good numerical results for known RM multiskyrmions, except $B = 1, 2$. The energy of skyrmion for rational map ansatz [15] in universal units $3\pi^2 F_\pi/e$ is:

$$M = \frac{1}{3\pi} \int \left\{ A_N r^2 f'^2 + 2B s_f^2 (f'^2 + 1) + \mathcal{I} \frac{s_f^4}{r^2} \right\} dr \quad (26)$$

The coefficient $A_N = 2(N-1)/N$ for symmetry group $SU(N)$ [34]. The quantity \mathcal{I} for $SU(2)$ -case is given in Appendix A, the inequality takes place $\mathcal{I} \geq B^2$. Direct numerical calculations have shown, and the analytical treatment here supports, that at large B and, hence, large \mathcal{I} multi-skyrmion looks like a spherical ball with profile equal to $F = \pi$ inside and $F = 0$ outside. The energy and B -number density of this configuration is concentrated at its boundary, similar to the domain walls system considered in [35] in connection with cosmological problems.

Consider such large B skyrmion within the "inclined step" approximation. Let W be the width of the step, and r_0 - the radius of the skyrmion where the profile $f = \pi/2$. $f = \pi/2 - (r - r_0)\pi/W$ for $r_0 - W/2 \leq r \leq r_0 + W/2$. Note that this approximation describes the usual domain wall energy [35] with accuracy $\sim 9\%$.

We write the energy in terms of W, r_0 , then minimize it with respect to both of these parameters, and find the minimal value of energy.

$$M(W, r_0) = \frac{\pi^2}{W}(B + A_N r_0^2) + W \left(B + \frac{3\mathcal{I}}{8r_0^2} \right) \quad (27)$$

This gives

$$W_{min} = \pi \left[\frac{B + A_N r_0^2}{B + 3\mathcal{I}/(8r_0^2)} \right]^{1/2} \quad (28)$$

and, after minimization, $r_{0min}^2 = \sqrt{3\mathcal{I}/(8A_N)}$. In dimensional units $r_0 = (6\mathcal{I}/A_N)^{1/4}/(F_\pi e)$. Since $\mathcal{I} \geq B^2$, the radius of minimized configuration grows as \sqrt{B} , at least. $W_{min} = \pi$, i.e. it does not depend on B for any $SU(N)$. The energy

$$M_{min} \simeq (2B + \sqrt{3A_N\mathcal{I}/2})/3 \quad (29)$$

For $SU(2)$ model $A_N = 1$ and the energy $M_{min} = (2B + \sqrt{3\mathcal{I}/2})/3$ should be compared with the lower bound $M_{LB} = (2B + \sqrt{\mathcal{I}})/3$. The formula gives the numbers for $B = 3, \dots, 22$ in remarkably good agreement with calculation within RM approximation, within 2 - 3% [7].

It is not difficult to calculate the corrections to these expressions, of relative order $1/B, 1/B^2, \dots$:

$$M(W, r_0) \simeq \frac{\pi^2}{W}(B + A_N r_0^2) + W \left[B(1 + \beta) + \frac{3\mathcal{I}}{8r_0^2}(1 + \gamma) \right], \quad (30)$$

$$\beta = \pi^2/(12B), \quad \gamma = (2\pi^2 + 17)/\sqrt{24\mathcal{I}}.$$

$$M_{min} \simeq [2B(1 + \beta/2) + \sqrt{3\mathcal{I}/8}(1 + \gamma/2)] \quad (31)$$

However, the first order correction in W does not improve the description of masses, and summation of all terms seems to be required.

So, we see that a very simple approximation provides a confirmation of a picture from numerical calculation of RM skyrmions as a two-phase object, a spherical ball with profile $f = \pi$ inside and $f = 0$ outside, and a fixed width envelope with fixed surface energy density, $\rho_M = M/(4\pi r_0^2) \simeq (2B + \sqrt{3\mathcal{I}/2})/(12\pi r_0^2)$. At large B $\rho_M \rightarrow Const$, but average mass density over the volume $\rightarrow 0$.

Consider also the influence of the mass term which gives the contribution

$$M.t. = \tilde{m} \int r^2(1 - \cos F)dr, \quad (32)$$

$\tilde{m} = 8m_\pi^2/(3\pi F_\pi^2 e^2)$. For strangeness, charm, or bottom the masses m_K, m_D or m_B should be inserted instead of m_π . In the "inclined step" approximation we obtain:

$$M.t. \simeq \tilde{m} \left[\frac{2}{3}r_0^3 + O(W^2) \right] \quad (33)$$

In view of this structure of the mass term it makes no influence on the width of the step W in lowest order, but the dimension of the soliton r_o becomes smaller:

$$r_o \rightarrow r_o - \tilde{m} \frac{r_o^2(B + A_N r_o^2)}{4\pi B}. \quad (34)$$

As it was expected from general grounds, dimensions of the soliton decrease with increasing \tilde{m} . However, even for large value of \tilde{m} the structure of multiskyrmion at large B remains the same: the chiral symmetry broken phase inside of the spherical wall where the main contribution to the mass and topological charge is concentrated. The behaviour of the energy density for $B = 22$ at different values of μ is shown in Fig. The value of the mass density inside of the ball is defined completely by the mass term with $1 - c_f = 2$. The baryon number density distribution is quite similar, with only difference that inside the bag it equals to zero. It follows from these results that RM approximated multiskyrmions cannot model real nuclei at large B , probably $B > 12 - 20$, and configurations like skyrmion crystals may be more valid for this purpose.

Besides the simple one-shell configurations considered in [7, 15] and here, multi-shell configurations can be of interest. Some examples of two-shell configurations with $B = 12, 13, 14$ have been considered recently [36]. The profile $f = 2\pi$ at $r = 0$ for such configurations and decreases to $f = 0$ for $r \rightarrow \infty$. We can also model such two-shell configuration in the domain-wall, or spherical bag approximation with a result

$$M \simeq (2B_1 + \sqrt{3\mathcal{I}_1/2})/3 + (2B_2 + \sqrt{3\mathcal{I}_2/2})/3, \quad (35)$$

with total baryon number $B = B_1 + B_2$. The profile f decreases from 2π to π in the first shell, and from π to 0 in the second. The radii of both shells should satisfy the condition $r_0^{(2)} \geq r_0^{(1)} + W$, so external shell should be large enough, with baryon number B_2 of several tens, at least. Since the ratio \mathcal{I}/B^2 is greater for smaller B , the energy (35) is greater than the energy of one-shell configuration considered before. Calculations performed in [36] also did not give results better than for one-shell configurations. However, more refined consideration would be of interest. Observation concerning the structure of large B multiskyrmions made here can be useful in view of possible cosmological applications of Skyrme-type models.

5 Concluding remarks

Here we have restricted ourselves with the Skyrme model and its straightforward extensions. However, many of the result are valid in other variants of the model: in the model with solitons stabilized by the explicit vector (ω) meson, or stabilized by the baryon number density squared, in the chiral perturbation theory, etc, see discussion in [14]b. The $B = 2$ torus-like configuration has been obtained within these models, as well as in the chiral quark-meson model [28], and it would be of interest to check if there are also multiskyrmions with $B \geq 3$.

We did not discuss a special classes of $SU(3)$ skyrmions, $SO(3)$ solitons and the problems of their observation, as well as $SU(3)$ skyrmion molecules. The discussion of these topics can be found in several papers of [12, 13]. Some new solutions which are not $SU(2)$ embeddings in $SU(3)$ or $SU(n)$ have been found in [34].

To conclude, the study of some processes, also at intermediate energies which, to some extent, are out of fashion now, can provide a very important check of fundamental principles and concepts of the elementary particles theory including the confinement of quarks and gluons. The confirmation of the predictions of the chiral soliton approach would provide

qualitatively new understanding of the origin of nuclear forces. If the existence of low energy radiatively decaying dibaryons is reliably established, it will change the long standing belief that nuclear matter fragments should consist necessarily of separate nucleons bound by their interactions. Therefore, the confirmation and checking of the results of experiments on dibaryons production, as well as of production of fragments of flavoured matter is extremely important. It would be possible at accelerators of moderate energies, like COSY (Juelich, FRG), KEK (Japan), Moscow meson factory (Troitsk, Russia), ITEP (Moscow), and some others. The production of multistrange states, as well as states with charm or bottom quantum numbers, is possible in heavy ion collisions, and also on accelerators like Japan Hadron Facility to be built in the near future.

The multiple flavour production realized in the production of flavored multibaryons possible, e.g., in heavy ion collisions, demands higher energy, of course, but multiple interaction processes and normal Fermi motion of nucleons inside of nuclei make effective thresholds much lower [37]. Studies of such flavoured multibaryons production would allow more complete and reliable checking of the model predictions.

We note finally that the low energy dibaryons have been obtained recently in [38] using the quantization procedure different from our.

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Appendix A. Inertia tensors of multiskyrmions.

The lagrangian density of the $SU(2)$ Skyrme model is given by

$$\mathcal{L} = -\frac{F_\pi^2}{16} \text{Tr} L_\mu L_\mu + \frac{1}{32e^2} \text{Tr} G_{\mu\nu}^2 + \frac{F_\pi^2 m_\pi^2}{16} \text{Tr}(U + U^\dagger - 2), \quad (\text{A1})$$

$L_\mu = \partial_\mu U U^\dagger$ is left chiral derivative, $L_\mu = iL_{\mu,k} \tau_k$, τ_k are Pauli matrices. $G_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu$ - strength of the chiral field. The Wess-Zumino term present in the action has been discussed in details in [13], and we shall omit this discussion here.

First we give the expression for the energy of $SU(2)$ - skyrmion as a function of profile F and unit vector \vec{n} , which is especially useful in some cases. Using definition $U = c_f + i s_f \vec{n} \vec{\tau}$, and relation

$$L_{\mu,k} L_{\nu,k} = \partial_\mu f \partial_\nu f + s_f^2 \partial_\mu \vec{n} \partial_\nu \vec{n}, \quad (\text{A2})$$

we obtain

$$M_{stat} = \int \left\{ \frac{F_\pi^2}{8} [(\vec{\partial}f)^2 + s_f^2 (\vec{\partial}n_i)^2] + \frac{s_f^2}{4e^2} \left[2[\vec{\partial}f \vec{\partial}n_i]^2 + s_f^2 [\vec{\partial}n_i \vec{\partial}n_k]^2 \right] + \rho_{M.t.} \right\} d^3r. \quad (\text{A3})$$

For the Ansatz based on rational maps the profile F depends only on variable r , and components of vector \vec{n} - on angular variables θ, ϕ . $n_x = (2 \text{Re } R)/(1 + |R|^2)$, $n_y = (2 \text{Im } R)/(1 + |R|^2)$, $n_z = (1 - |R|^2)/(1 + |R|^2)$, where R is a rational function of variable $z = tg(\theta/2) \exp(i\phi)$ defining the map from $S^2 \rightarrow S^2$. In this case the gradients of functions F and \vec{n} are orthogonal (recall that $\vec{\partial}_r = \vec{n}_r \partial_r + \vec{n}_\theta \partial_\theta / r + \vec{n}_\phi \partial_\phi / (r s_\theta)$ $\vec{n}_r = \vec{r}/r = (s_\theta c_\phi, s_\theta s_\phi, c_\theta)$, $\vec{n}_\theta = (-c_\theta c_\phi, -c_\theta s_\phi, s_\theta)$, $\vec{n}_\phi = (s_\phi, -c_\phi, 0)$) and $[\vec{\partial}f \vec{\partial}n_1]^2 = f'^2 (\vec{\partial}n_1)^2$, etc. Taking into account relations

$$n_3^2 [\vec{\partial}n_2 \vec{\partial}n_3]^2 = n_1^2 [\vec{\partial}n_1 \vec{\partial}n_2]^2, \quad n_3^2 [\vec{\partial}n_1 \vec{\partial}n_3]^2 = n_2^2 [\vec{\partial}n_1 \vec{\partial}n_2]^2, \quad (\text{A4})$$

one can present (A3) as

$$M_{stat} = \int \left\{ \frac{F_\pi^2}{8} [(f')^2 + s_f^2 (\vec{\partial}n_i)^2] + \frac{s_f^2}{2e^2} \left[f'^2 (\vec{\partial}n_i)^2 + s_f^2 [\vec{\partial}n_1 \vec{\partial}n_2]^2 / n_3^2 \right] + \rho_{M.t.} \right\} d^3r. \quad (\text{A5})$$

Usually the notation is introduced

$$\mathcal{I} = \frac{1}{4\pi} \int r^4 \frac{[\vec{\partial}n_1 \vec{\partial}n_2]^2}{n_3^2} d\Omega = \frac{1}{4\pi} \int \left(\frac{(1+|z|^2)}{(1+|R|^2)} \frac{|dR|}{|dz|} \right)^4 \frac{2idz d\bar{z}}{(1+|z|^2)^2}, \quad (\text{A6})$$

and using the equation

$$\int r^2 (\vec{\partial}n_k)^2 d\Omega = 2 \int r^2 \frac{[\vec{\partial}n_1 \vec{\partial}n_2]}{|n_3|} d\Omega = 2 \int \frac{2idRd\bar{R}}{(1+|R|^2)^2} = 8\pi\mathcal{N} \quad (\text{A7})$$

obtain finally

$$M_{stat} = 4\pi \int \left\{ \frac{F_\pi^2}{8} (f'^2 r^2 + 2s_f^2 \mathcal{N}) + \frac{s_f^2}{2e^2} \left[2f'^2 \mathcal{N} + s_f^2 \mathcal{I}/r^2 \right] + r^2 \rho_{M.t.} \right\} dr. \quad (\text{A8})$$

To find the minimal energy configuration at fixed $\mathcal{N} = B$ one minimizes \mathcal{I} , and then finds the profile $F(r)$ by minimizing energy (A8).

To quantize zero modes one uses ansatz $U(t, r) = A(t)U(O_{ik}(t)r_k)A^\dagger(t)$, and evident relation

$$\partial_i U = \dot{U} = \dot{A}U(\vec{r}(t))A^\dagger + AU(\vec{r}(t))\dot{A}^\dagger + \dot{r}_i(t)A\partial_i U(\vec{r}(t))A^\dagger, \quad (\text{A9})$$

where $r_i(t) = O_{ik}(t)r_k$ is body-fixed coordinate.

Angular velocities of spatial (or orbital) rotations are introduced according to:

$$\dot{r}_i = \dot{O}_{ik}r'_k = \dot{O}_{ik}O_{kl}^{-1}r_l(t) = -\epsilon_{ilm}\Omega_m r_l(t)$$

and integration is performed in coordinate system bound to soliton (body-fixed).

The rotation, or zero-mode energy of $SU(2)$ skyrmions as a function of angular velocities is

$$E_{rot} = \frac{1}{2}\Theta_{ab}^I \omega_a \omega_b + \Theta_{ab}^{int} \omega_a \Omega_b + \frac{1}{2}\Theta_{ab}^J \Omega_a \Omega_b. \quad (\text{A10})$$

The isotopical tensor of inertia for arbitrary $SU(2)$ skyrmion is:

$$\Theta_{ab}^I = \int s_f^2 \left\{ (\delta_{ab} - n_a n_b) \left(\frac{F_\pi^2}{4} + \frac{(\vec{\partial}f)^2}{e^2} \right) + \frac{s_f^2}{e^2} \partial_l n_a \partial_l n_b \right\} d^3r. \quad (\text{A11})$$

For the RM ansatz trace of this tensor of inertia is

$$\Theta_{aa}^I(RM) = 4\pi \int s_f^2 \left\{ \frac{F_\pi^2}{2} + \frac{2}{e^2} \left(f'^2 + \mathcal{N} \frac{s_f^2}{r^2} \right) \right\} r^2 dr. \quad (\text{A12})$$

The orbital inertia tensor gives contribution to the energy $\Theta_{ab}^J \Omega_a \Omega_b / 2$, and for arbitrary configuration using the same notations is given by:

$$\Theta_{ab}^J = \int \left\{ \frac{F_\pi^2}{4} (\partial_i f \partial_k f + s_f^2 \partial_i \vec{n} \partial_k \vec{n}) + \frac{s_f^2}{e^2} \left[\partial_i f \partial_k f (\vec{\partial}n_l)^2 + (\vec{\partial}f)^2 \partial_i \vec{n} \partial_k \vec{n} - \partial_i f \partial_l f \partial_l \vec{n} \partial_k \vec{n} - \partial_k f \partial_l f \partial_l \vec{n} \partial_i \vec{n} + s_f^2 [(\vec{\partial}n_l)^2 \partial_i \vec{n} \partial_k \vec{n} - (\partial_i \vec{n} \partial_l \vec{n})(\partial_k \vec{n} \partial_l \vec{n})] \right] \right\} \epsilon_{i\alpha\alpha} \epsilon_{k\beta\beta} r_\alpha r_\beta d^3r. \quad (\text{A13})$$

This expression can be simplified for RM ansatz:

$$\Theta_{ab}^J = \int s_f^2 \left\{ \left[\frac{F_\pi^2}{4} + \frac{f'^2}{e^2} + \frac{s_f^2}{e^2} (\vec{\partial}n_l)^2 \right] \left[(\vec{\partial}n_l)^2 (r^2 \delta_{ab} - r_a r_b) - \partial_a \vec{n} \partial_b \vec{n} r^2 \right] - \right.$$

$$-\frac{s_f^2}{e^2} \left[(\partial_i \vec{n} \partial_k \vec{n}) (\partial_i \vec{n} \partial_k \vec{n}) (r^2 \delta_{ab} - r_a r_b) - r^2 (\partial_a \vec{n} \partial_l \vec{n}) (\partial_b \vec{n} \partial_l \vec{n}) \right] \Big\} d^3 r \quad (A14)$$

It allows to obtain easily the trace of the inertia tensor Θ_{aa}^J .

$$\Theta_{aa}^J(RM) = 4\pi \int s_f^2 \left\{ \frac{F_\pi^2}{2} \mathcal{N} + \frac{2}{e^2} \left(f'^2 \mathcal{N} + \mathcal{I} \frac{s_f^2}{r^2} \right) \right\} r^2 dr. \quad (A15)$$

It is easy to establish the inequality for traces of isotopical and orbital tensors of inertia:

$$\Theta_{aa}^J - B\Theta_{aa}^I = \frac{8\pi}{e^2} (\mathcal{I} - B^2) \int s_f^4 dr \geq 0, \quad (A16)$$

since $\mathcal{I} \geq B^2$, $\mathcal{N} = B$. The interference (mixed) tensor of inertia which defines also the isovector part of the magnetic transition operator equals to:

$$\begin{aligned} \Theta_{ab}^{int} = \int s_f^2 \left\{ \left[\frac{F_\pi^2}{4} + \frac{1}{e^2} [(\partial_\nu f)^2 + s_f^2 (\partial_\nu \vec{n})^2] \right] \partial_i n_l - \right. \\ \left. - \frac{1}{e^2} (\partial_i f \partial_\nu f + s_f^2 \partial_i \vec{n} \partial_\nu \vec{n}) \partial_\nu n_l \right\} n_k \epsilon_{kla} \epsilon_{i\alpha b} r_\alpha d^3 r. \end{aligned} \quad (A17)$$

The components of spatial angular velocities interfere with components $\omega_1, \omega_2, \omega_3$ of angular velocities of rotation in configuration space, only.

Numerically the components of the mixed tensor of inertia are much smaller than those of isotopical, or orbital tensor of inertia, except special cases of "hedgehogs" when $-\Theta^{int} = \Theta^I = \Theta^J$, and axially symmetrical configurations when for the 3-d components of inertia relations hold $-\Theta_{33}^{int} = n\Theta_{33}^I = \Theta_{33}^J/n$.

Note, that most general formulas for tensors of inertia are presented here for the first time. For the case of *RM* configurations they differ in some details from those given in the literature.

Appendix B. Electromagnetic transition operators.

Here we prove, for completeness, in general form some statements concerning isovector (octet in *SU(3)* case) vector charge and isovector magnetic momentum operator.

There is the following connection between isovector current and isospin generator

$$V_{0,a} = \frac{1}{2} Tr(A^\dagger \lambda_a A \lambda_b) I_b^{bf} = R_{ab}(A) I_b^{bf}, \quad (B1)$$

where the isospin generator in body-fixed (connected with soliton) coordinate system is

$$I_b^{bf} = \partial L^{rot}(\omega, \Omega) / \partial \omega_b. \quad (B2)$$

$a, b = 1, 2, 3$ for *SU(2)*-model, and $a, b = 1, \dots, 8$ for *SU(3)*-model. To prove this consider ansatz

$$U = e^{-i\alpha_a \lambda_a / 2} A(t) U_0 A^\dagger(t) e^{i\alpha_a \lambda_a / 2} \quad (B3)$$

The vector Noether current is a coefficient before derivative of the probe function, $\partial_\mu \alpha$. In the lowest order in α we obtain for the chiral derivative:

$$U^\dagger \partial_0 U = A [U_0^\dagger A^\dagger (\dot{A} - i\dot{\alpha} A / 2) U_0 - A^\dagger (\dot{A} - i\dot{\alpha} A)] A^\dagger \quad (B4)$$

Using the definition of angular velocities of rotation in configuration space ω_a , we obtain

$$A^\dagger \dot{A} - iA^\dagger \dot{\alpha} A / 2 = -\frac{i}{2} \lambda_b (\omega_b + R_{ab}(A) \dot{\alpha}_a), \quad (B5)$$

where real orthogonal matrix

$$R_{ab}(A) = \frac{1}{2} \text{Tr}(A^\dagger \lambda_a A \lambda_b). \quad (B6)$$

Since the dependence on $\dot{\alpha}$ reduces to simple addition to angular velocity according to (B5), formula (B1) follows immediately.

According to the well known relation,

$$Q = B + I_3/2 = B + V_{0,3}/2 \quad (B7)$$

the baryonic (topological) charge and $3-d$ component of the isospin generator contribute to the charge of the quantized skyrmion.

We prove also that there is simple connection between isovector (octet for $SU(3)$ model) magnetic momentum operator of the skyrmion and mixed (interference) tensor of inertia. Note first that the lagrangian of arbitrary chiral model, not only Skyrme model, because of Lorentz invariance can be presented as a sum, with some coefficients, of contributions of the type:

$$\mathcal{L}_{M,N} = \text{Tr}(U^\dagger \dot{U} M U^\dagger \dot{U} N - U^\dagger \partial_k U M U^\dagger \partial_k U N), \quad (B8)$$

where M and N are some matrices. E.g., for the second order term $M = N = 1$. The contribution into rotational energy, proportional to Ω , ω , which comes from the first term in (B8) and defines mixed or interference tensor of inertia is (see (A9) above):

$$\Theta_{ab}^{int} \omega_a \Omega_b = \int \text{Tr}(U_0^\dagger A^\dagger \dot{A} U_0 - A^\dagger \dot{A}) \tilde{M} U_0^\dagger \partial_k U_0 \tilde{N} \dot{r}_k d^3 r + (M \rightarrow N), \quad (B9)$$

$\tilde{M} = A^\dagger M A$, $\tilde{N} = A^\dagger N A$. Or,

$$\Theta_{ab}^{int} = -\frac{i}{2} \epsilon_{bjk} \int r_j(t) \text{Tr}(U_0^\dagger \lambda_a U_0 - \lambda_a) \tilde{M} U_0^\dagger \partial_k U_0 \tilde{N} d^3 r + (M \rightarrow N) \quad (B10)$$

$r_j(t)$ and ∂_k are body-fixed here. From the second term in expression (B8) we obtain for the spatial components of the vector current:

$$V_k^a = \frac{i}{2} \text{Tr}(U_0^\dagger A^\dagger \lambda_a A U_0 - A^\dagger \lambda_a A) \tilde{M} U_0^\dagger \partial_k U_0 \tilde{N} + (M \rightarrow N) \quad (B11)$$

Taking into account that $A^\dagger \lambda_a A = R_{ab}(A) \lambda_b$, $R_{ab} = \frac{1}{2} \text{Tr} A^\dagger \lambda_a A \lambda_b$ and $\partial_k = O_{lk} \partial_l^{bf}$,

$$V_k^a = \frac{i}{2} R_{ab} O_{lk} \text{Tr}(U_0^\dagger \lambda_b U_0 - \lambda_b) \tilde{M} U_0^\dagger \partial_l U_0 \tilde{N} + (M \rightarrow N) \quad (B12)$$

By definition

$$\mu_i^a = \frac{1}{2} \epsilon_{ijk} \int r_j V_k^a d^3 r, \quad (B13)$$

or

$$\mu_i^a = \frac{i}{4} \epsilon_{ijk} R_{ab}(A) O_{qk} O_{pj} \int r_p(t) \text{Tr}(U_0^\dagger \lambda_b U_0 - \lambda_b) \tilde{M} U_0^\dagger \partial_q U_0 \tilde{N} + (M \rightarrow N). \quad (B14)$$

Taking into account that

$$\epsilon_{ijk} O_{pj} O_{qk} = \epsilon_{pql} O_{li}$$

we obtain the desired relation between components of the magnetic momentum operator and mixed tensor of inertia in the body-fixed coordinate system:

$$\mu_i^a = -\frac{1}{2} R_{ab}(A) \Theta_{bl}^{int} O_{li}. \quad (B15)$$

In some particular cases this relation was used previously [8, 10].

For the transition matrix elements calculations it is necessary to average this expression over wave functions of some initial and final states, see Section 2.

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Figure caption.

The mass density distribution of the rational map multiskyrmion with $B = 22$ as a function of the distance from center of skyrmion for different values of mass in the chiral symmetry breaking term .

a) pion mass in the mass term, b) kaon mass, c) D -meson mass, the mass density is divided by 10.